107學年度第2學期土研所博士班資格考試(108.03)

有限元素法

(1) A simply-supported, elastic Timoshenko beam is considered as shown in Fig. 1(a), in which L, A, E and I denote the length, cross-sectional area, Young's moudulus of the beam, and its second moment of inertia of the cross-sectional area, respectively. The beam is subjected to a transverse load $q(x)$ in its domain ($0<x<L$). In Timoshenko's beam theory, the transverse shear rotation (strain) is assumed to be constant through the thickness, and the assumed displacement field is thus given as (see Fig. 1(b))

$$u(x,z) = -z\psi(x) \tag{1a}$$

$$w(x,z) = w(x) \tag{1b}$$

where $\psi = (dw/dx) - \beta$, and dw/dx = the bending rotation and $\beta(x)$ = the shear rotation; $w(x)$ is the mid-plane displacement in the z direction.

In order to recognize the nonuniform shear stress distribution at a section, we modify the average shear stress by introducing a factor λ, as follows.

$$\tau_{xz} = \lambda G \beta \tag{1c}$$

where λ =the correction factor and G=the shear modulus.

The bending moment M and shear force Q are defined as

$$M = -\int_A \sigma_x z\,dA, \quad \text{and} \quad Q = -\int_A \tau_{xz}\,dA. \tag{1d}$$

The potential energy functional of a Timoshenko beam is given as follows:

$$\Pi_p(\psi, w) = \int_0^L \left[\frac{EI}{2}\left(\frac{d\psi}{dx}\right)^2 + \frac{\lambda GA}{2}\left(\frac{dw}{dx} - \psi\right)^2 - qw \right] dx\,, \tag{1e}$$

where ψ and w are regarded as the primary variables subject to variation.

(a) Prove that the potential energy functional of a Timoshenko beam is identical to Eq. (1e). (10%)

(b) Explain why the mid-plane displacement u_0, in which $u(x,0) = u_0$, can be discarded from the displacement field (i.e., Eq. (1a). (10%)

(c) Determine the weak form equivant to the ODE and NBC. (10%)

(d) Derive the Euler-Lagrange equations and the associated boundary conditions. (10%)

(e) Determine the exact solution of displacement function $w(x)$ when $q(x) = q_0 \sin(\pi x / L)$. (10%)

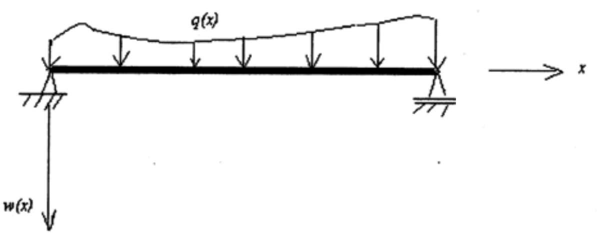

Fig. 1(a) An elastic Timoshenko beam

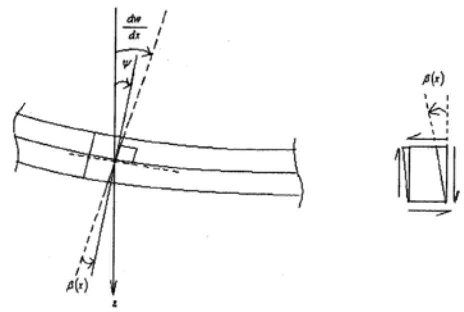

Fig. 1(b) The elastic deformation of a typical Timoshenko beam element.

(2) If the nodal values of the element shown in Fig. 2 are $u_i = \hat{u}_i \ (i=1, 2, 3)$, compute u, $\partial u/\partial x$ and $\partial u/\partial y$ at point $(x, y) = (0.325, 0.325)$. (25%)

Hint: The linear interpolation functions $\phi_i^{(e)}(x, y) = \frac{1}{2A_e}\left(\alpha_i^{(e)} + \beta_i^{(e)} x + \gamma_i^{(e)} y\right) \ (i=1, 2, 3)$ and $\alpha_i^{(e)} = x_j y_k - x_k y_j$, $\beta_i^{(e)} = y_j - y_k$, $\gamma_i = -(x_j - x_k)$, ($i \neq j \neq k$; i, j and k permute in a natural order).

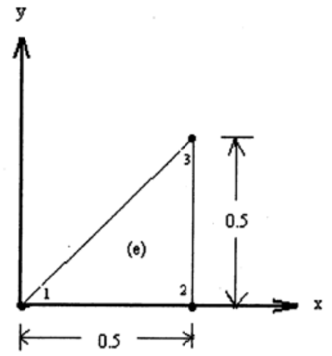

Fig. 2

(3) Show that the bilinear interpolation functions for the four-node triangular element in Fig. 3 are in the form of (25%)

$$\phi_i^{(e)} = A_i + B_i \xi + C_i \eta + D_i \xi \eta \quad (i=1, 2, 3, 4)$$

where $A_1 = 1$, $A_2 = A_3 = A_4 = 0$, $-B_1 = B_2 = 1/a$, $B_3 = B_4 = 0$,

$$C_1 = \frac{6ab - a^2 - 2b^2}{ac(a-2b)}, \quad C_2 = \frac{2b(a+b)}{ac(a-2b)}, \quad C_3 = \frac{a+b}{c(a-2b)}, \quad C_4 = \frac{-9b}{c(a-2b)}$$

$$D_1 = D_2 = D_3 = -\frac{1}{3} D_4 = -\frac{3}{c(a-2b)}.$$

Hint: The coordinate of point 4 is $\left(\frac{a+b}{3}, \frac{c}{3}\right)$.

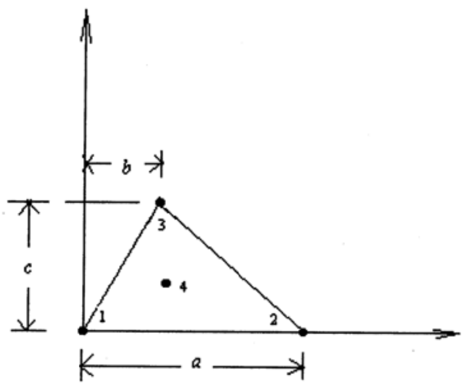

Fig. 3

107 學年度第二學期博士學位候選人資格考
彈性力學
Theory of Elasticity

Time: 100 minutes
No electronic calculator or devices are allowed.

1. Show materials with cubic symmetry require three independent elastic constants. (25%)

2. Derive the Beltrami-Michell compatibility equations. (25%)

3. Solve the St. Venant torsion problem of a solid, long, prismatic cylinder with elliptic cross-section for its stress distribution under applied torque T. (25%)

4. Derive the tensile strength of brittle materials measured from the Brazilian tension test to be $\sigma=2P/(\pi D)$, where P is the diametrical compressive loading and D the diameter of the specimen. What is the physical units of σ? Why is specimen length not involved in calculating the strength? (25%)

Pavement Analysis

National Cheng Kung University

Qualification Exam for Ph.D. Students

29 March, 2019

1. What is the difference in the way flexible and rigid pavements carry load? (25 points)

2. It is recognized that factors affecting pavement design include traffic, environment, materials, and failure criteria. All these factors vary from one area to the other area. How are you going to categorize these factors into different zones so that proper parameters for pavement analysis can be applied to that particular zone? (25 points)

3. Use the 1993 AASHTO flexible pavement design procedure to determine the lowest initial cost pavement section for the conditions and materials given below. Do not consider the cost of excavation in your analysis. Present your answer by drawing the final cross-section of the pavement on top of the subgrade. (50 points)

   The section is a fill section, which requires that the final elevation of the pavement be 54 inches above the level of the existing subgrade. The pavement will be built on a mountain near Taiwan. You are to design the section to provide full frost protection based on the average of the three coldest years in thirty years. Details on design inputs and traffic information are given on the following page.

   You should consider the following combinations of materials:

   - Full-depth asphalt concrete;
   - Asphalt concrete and crushed stone;
   - Asphalt concrete, crushed stone, and pit-run gravel;
   - Asphalt concrete and cement stabilized base;
   - Asphalt concrete, cement stabilized base, and crushed stone;
   - Asphalt concrete, cement stabilized base, crushed stone, and pit-run gravel.

Material properties and costs are given below. Note that you may be able to eliminate one or more of these combinations on the basis of cost.

Construction	Material Properties	Cost $/sq.yd.-in
Asphalt Concrete Modulus (68 F)	300,000 psi	1.50
Bituminous Stabilized Base Modulus	200,000 psi	1.00
Crushed Stone Resilient Modulus	25,000 psi	0.40
Pit-run Gravel Resilient Modulus	12,000 psi	0.32

Based on a knowledge of the granular materials properties (gradation, permeability, etc.), the construction site location, and the probable location of the materials in the pavement structure, it was determined that the crushed stone has good drainage characteristics and will be saturated less than 5% of the time. The put-run gravel was determined to have fair drainage characteristics and will be saturated as much as 25% of the time. Both materials are non-frost susceptible and during the frost season, the materials are expected to be frozen at l0% moisture content. Both materials have dry density of about 103 pcf.

Roadbed Soil	Season	Resilient Modulus
mid-December to late February	Winter	30,000 psi
early March to Late April	Spring	1,000 psi
early may to mid-December	Summer, Fall	5,000 psi

Four-lane Highway

The following design inputs were selected based on consideration of pavement functional importance, knowledge of local construction quality, and engineering experience:

The data below were obtained on a highway with 3,146 trucks considered to be representative of the traffic to be representative of the traffic to be carried by the new highway. All vehicle types are represented in the data set. Ignore the effect of all other vehicles in your computations. The following data were used to calculate the truck factor on this highway.

Performance Period	20 years
Design Reliability	90 %
Overall Standard Deviation (including traffic)	0.45
Initial (Terminal) Pavement Serviceability Index	4.5 (2.5)
Traffic: Initial ADT (two-way)	13,150 vehicles
Directional Distribution	50 % each way
Percent Trucks	20%
Percent Trucks in Outer Lane	85%
Growth Rate	5% per year

Axle Load Groups, lbs No. of Axles	Representative Axle Load, lbs	No. of Axles	LEF	ESAL
Single Axle				
Under 3,000	2,000	512		
3,000-6,999	5,000	536		
7,000-7,999	7,500	239		
8,000-11,999	10,000	1,453		
12,000-15,999	14,000	279		
16,000-18,000	17,000	106		
18,000-20,000	19,000	43		
20,000-21,999	21,000	4		
22,000-23,999	23,000	3		
Tandem				
Under 6,000	4,000	9		
6,000-11,999	9,000	337		
12,000-17,999	15,000	396		
8,000-23,999	21,000	457		
24,000-29,999	27,000	815		
30,000-32,000	31,000	342		
32,000-33,999	33,000	243		
34,000-35,999	35,000	173		
36,000-37,999	37,000	71		
38,000-39,999	39,000	9		
40,000-41,999	41,000	0		
42,000-43,999	43,000	1		

107 學年度第二學期博士學位候選人資格考試

高等瀝青材料學 (20% for each question)

1. Explain and comment the following paragraph.
   For the past fifty years or more, pavement design agencies have always pressed for an immediate answer to their needs and problems. Long-term, well considered sequential efforts usually have been rejected because of the time and expense involved. The effect has been that structural and mixture design procedures have stagnated, and an integrated design and analysis system have not evolved.

2. Briefly describe the asphalt cement grading system. (including PEN, AC, AR, and SHRP)

3. Briefly describe the aggregate properties tests for the Superpave mix design method.

4. Briefly describe the hot mix asphalt facilities.(including the drum mix facility and batch facility)

5. Briefly describe the properties of an ideal pavement binder.

107學年度第二學期博士學位候選人資格考試

工程統計

作答方式：Open Book　考試時間：100分鐘　及格分數：60分

1. A contractor estimates that the mean completion time of a job is 180 days, meanwhile, he is 90% confident to complete the job within 220 days. Assume the completion time of this job to be a **Gaussian** random variable.
   (a) Find the standard deviation of the completion time of this job. (10%)
   (b) Determine the probability of this job being finished within 240 days. (10%)

2. Strikes among construction workers occur according to the **Poisson** process, and on the average there is one strike every 2 years. The loss due to a strike is modeled by a **log-normal** distribution with mean $500,000 and coefficient of variation 20%.
   (a) What is the probability that the loss exceeds $400,000 during a strike? (10%)
   (b) If a job will take 3 years to complete, what is its expected loss from possible strikes? (10%)

3. Given 9 records from a **normal** distribution as follows:
   82, 75, 95, 90, 88, 92, 78, 85, 80.
   (a) Determine the 95% confidence interval for the mean. (10%)
   (b) Determine the 95% upper confidence limit for the variance. (10%)
   (c) The null and alternative hypotheses are

   $$\begin{cases} H_0 : \mu = 80 \\ H_1 : \mu \neq 80 \end{cases}.$$

   Test the hypothesis at the 5% significance level. (10%)

4. An empirical formula to evaluate Y with given X is proposed as follows:

   $$Y = aX^b.$$

   The observed data is given by $(x_i, y_i),\ i = 1, 2, \ldots, n$.
   (a) For performing the **linear regression** of $\ln Y$ on $\ln X$, find a formula for the sum of squared errors. (10%)
   (b) Based on the result of 4(a), find a formula to estimate the conditional variance $Var(\ln Y|\ln x)$. (10%)
   (c) Based on the three estimates of a, b, and $Var(\ln Y|\ln x)$, find the conditional probability $P(Y \leq d|x = c)$, and give your assumption. (10%)

107 學年度第二學期博士學位候選人資格考試

考試科目: 高等鋼結構

考試方式: Closed Book
考試時間: 100 分鐘

1. [24%] Please give detailed explanations for the following terminologies in steel structures.
   (a) HSLA steel (b) Bauschinger effect (c) W-shape (d) BH (e) RH
   (f) Shape Factor (g) Toughness (e) Ductility

2. [12%] Please list four factors affecting the buckling strength of a real steel column.

3. [15%] Please list five properties required for the "earthquake-resistant" steel?

4. [9%] Consider a square element subjected to the same in-plane shear stress (τ) at the four sides (i.e., pure shear condition) as shown in Fig. 1. This square element is made of steel with the tensile yield stress of F_y. Please derive the shear stresses (τ_y) to cause yield failure of this square element according to von Mises yield criterion. Show your answer of τ_y as the function of F_y.

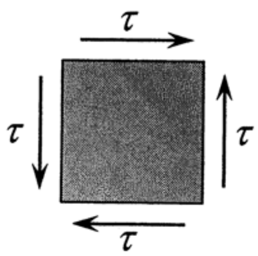

Fig. 1

5. [20%] Consider an initially-crooked and simply-supported elastic column with bending stiffness EI and length L subjected to an axial compressive force P as shown in Fig. 2. Assume the initial deflection function of this column is $v_0(x)=\Delta_0 \sin(\pi x/L)$ at P=0 and bending moment $M(x)$=0. Please derive the deflection function $v(x)$ of this column under axial compressive force P.

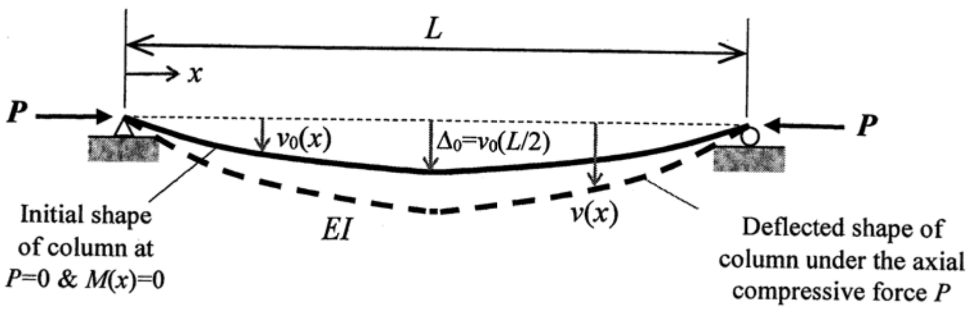

Fig. 2

6. [20%] Consider a T-shaped steel beam made of an A992 steel (F_y=50ksi) top flange and an A36 steel (F_y=36ksi) stem as shown in Fig.3. Please compute:
   (a) The first yield moment M_y of this beam.
   (b) The plastic moment M_p of this beam.
   (Note: Neglect flange local buckling & stem local buckling of the following steel beam.)

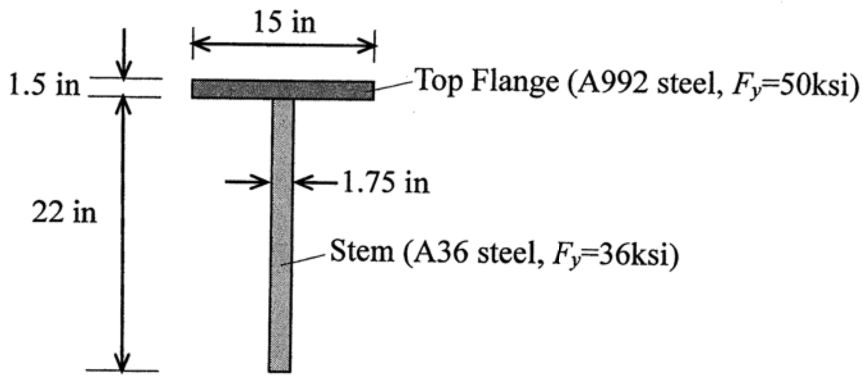

Fig. 3