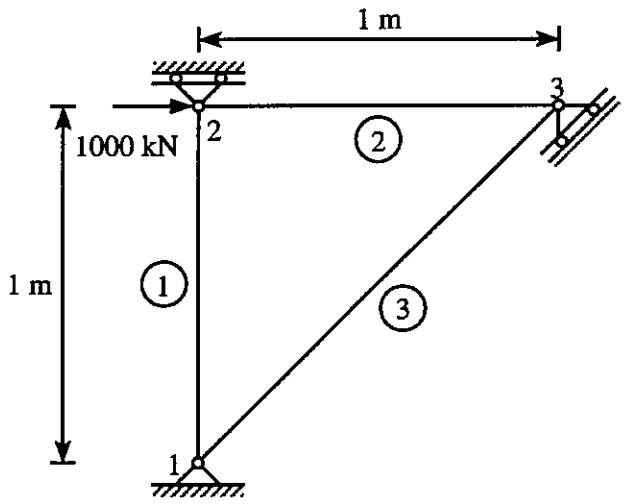
Finite Element Method

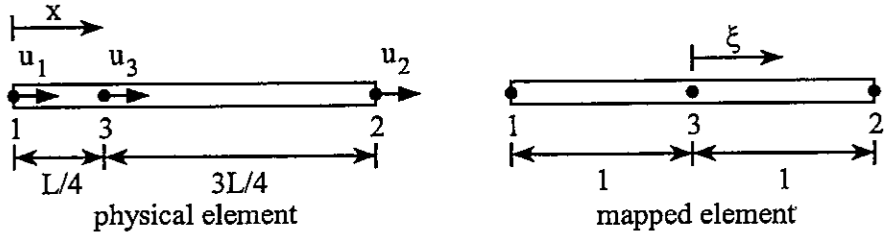
(Close book, 100 minutes, 70% to pass)

1. The following True-False questions refer to finite elements based on assumed displacements. (15%)
   () A. Within an element, the calculated stresses are more accurate than the displacements.
   () B. The equilibrium conditions are satisfied at every material point within the elements.
   () C. Arbitrarily assumed expressions for the strains always satisfy the compatibility within the elements.
   () D. Arbitrarily assumed expressions for the displacement always satisfy the compatibility within the elements, provided the displacements are single-valued and continuous.
   () E. The stiffness matrix [K] of a partial supported structure is non-singular (i.e. $|[K]| \neq 0$).

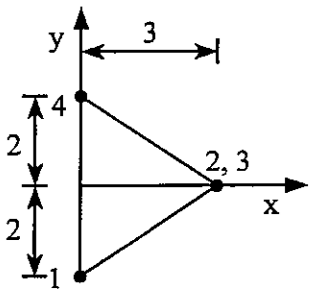
2. For the plane truss shown, determine the displacements and reactions at each node. Calculate the axial force in each member. Let E = 210 GPa. Assume $A = 6\ \text{cm}^2$ for elements 1 and 2, and $A = 6\sqrt{2}\ \text{cm}^2$ for element 3. (20%)



3. A 3-node isoparametrictruss element is shown below.Assume $\{\varepsilon_x\}_{1\times1} = [B]_{1\times3}\{d\}_{3\times1}$, where $\{d\}^T = \{u_1, u_3, u_2\}$. (i) Calculate the $[B]_{1\times3}$ matrix in terms of ξ for the physical element. (ii) Calculate the strain ε_x at node 1. Does this element behave good or bad? (15%)

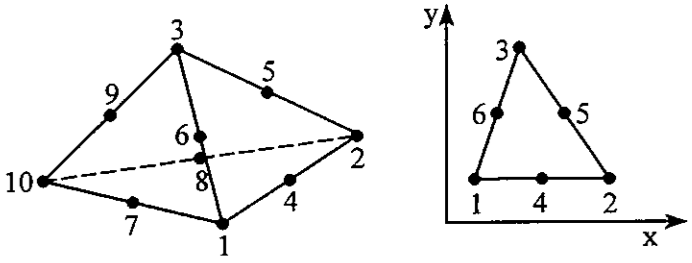


4. Compute the Jacobian matrix [J] and its determinant |J| for the element. (15%)

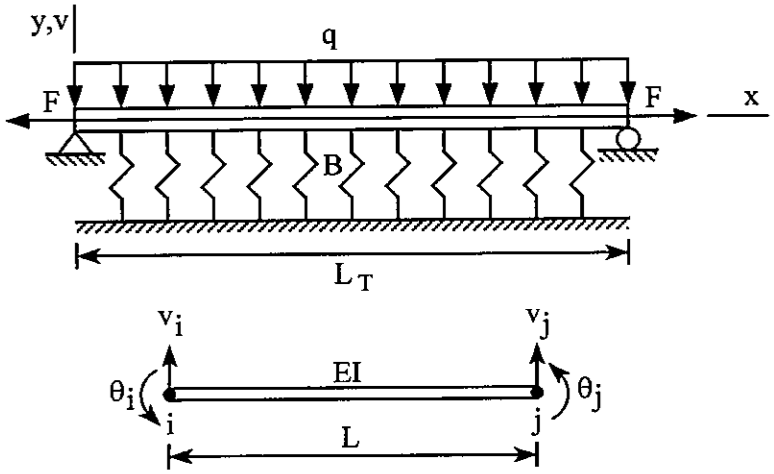


5. The face containing nodes 1-4-2-5-3-6 of a 10-node tetrahedron element coincides with z = 0 plane as shown. This face is subjected to uniform traction $p_z = p$ in z direction. Use volume coordinate or area coordinate to calculate the consistent loads in z direction for all 10 nodes. (20%)
Note: the formula for integration over triangle area A is

$$\int_A \xi_1^k \xi_2^m \xi_3^n dA = 2A \frac{k!m!n!}{(2+k+m+n)!}$$



6. A simply supported beam rested on elastic foundation (with foundation modulus B) is shown below. The beam is subjected to constant axial force F and uniformly distributed lateral load q. The differential equation of the beam can be written as: $EIv_{,xxxx} - q - Fv_{,xx} + Bv = 0$. Assume the beam is divided into N_{els} elements in the axial direction. Each beam element has the nodald.o.f $\{d\} = \{v_i, \theta_i, v_j, \theta_j\}^T$ and v(x) = [N]{d}. Use the Galerkin method and the beam element to formulate the expressionof the element stiffness matrix [k]. (15%)



Elasticity

Department of Civil Engineering, National Cheng Kung University

***Return question-paper after exam**
***Closed-book exam**

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1. **(30 points)** Given the relations

$$\sigma_{ij} = s_{ij} + \frac{1}{3}\sigma_{kk}\delta_{ij}, \quad J_2 = \frac{1}{2}s_{ij}s_{ji}, \quad J_3 = \frac{1}{3}s_{ij}s_{jk}s_{ki},$$

where σ and s are symmetric second-order tensors. Show that

(a) $s_{ii} = 0$.

(b) $J_{2,\sigma ij} = s_{ij}$.

(c) $J_{3,\sigma ij} = s_{ik}s_{kj} - 2J_2\delta_{ij}/3$.

2. **(30 points)** For isotropic linear elastic materials, prove the following relations between the elastic moduli E (Young's modulus), G (shear modulus), ν (Poisson's ratio) and k (bulk modulus):

$$\nu = \frac{3k - E}{6k}, \quad k = \frac{GE}{9G - 3E}.$$

Using the fact that for an isotropic body the principal axes of stress and strain coincide and assuming that the stress-strain relation is linear so that superposition holds, derive

$$\varepsilon_{ij} = \frac{1+\nu}{E}\sigma_{ij} - \frac{\nu}{E}\sigma_{mm}\delta_{ij},$$

directly from the definitions of E and ν given by

$$E = \frac{\sigma_{11}}{\varepsilon_{11}}, \quad \nu = -\frac{\varepsilon_{22}}{\varepsilon_{11}} = -\frac{\varepsilon_{33}}{\varepsilon_{11}},$$

in simple tension test.

3. **(30 points)** The elastic complementary energy density U^c is defined viz.

$$U^c \equiv \int_0^{\sigma_{ij}} \varepsilon_{ij} d\sigma_{ij}.$$

Based on this concept, the behavior of an isotropic nonlinear elastic material is described by the following assumed polynomial expression for U^c:

$$U^c\left(I_1, J_2, J_3\right) = aI_1^2 + bJ_2 + cJ_3^2,$$

where a, b and c are constants and I_1, J_2 and J_3 are the stress invariants defined in Problem

1 of this exam.

(a) Derive the stress-strain relations for this material in terms of the constants a, b and c.

(b) Derive the stress-strain relations in simple tension. Find an expression for the tangent Young's modulus E_t defined viz.

$$E_t \equiv \frac{d\sigma}{d\varepsilon},$$

in simple tension, in terms of the stress σ. What is the value of the initial tangent Young's modulus $E_t(0)$ (at $\sigma = 0$)?

(c) Show that the constitutive equations are reduced to those of the isotropic linear elastic material for $c = 0$. Find the relations between the constants a, b, and the elastic moduli E, ν for this case.

4. **(10 points)** For isotropic materials, show that the principal axes of strain coincide with the principal axes of stress. Further, show that the principal stresses can be expressed in terms of the principal strains via

$$\sigma_i = 2\mu\varepsilon_i + \lambda\varepsilon_{kk},$$

where σ_i and ε_i are the principal stresses and strains, respectively.

Qualified Examination of Doctorate Program in

Department of Civil Engineering, NCKU

SOIL MECHANICS

A. Describe the following terminologies, (20 points)

1. Consistency Index
2. Diffuse Double Water
3. Activity
4. Plasticity Chart

B. Derive the equivalent hydraulic conductivity in stratified soil with the flow in the vertical direction. (Sketch a diagram if any), (20 points)

C. The logarithm-of-time method and the square root-of-time method are two methods which used in common, to determine the coefficient of consolidation. List the advantages and limitations of these two methods.
(30 points)

D. A consolidated-undrained test on a normally consolidated clay yielded the following results: (30 points)

$\sigma_3 = 84\ kN/m^2$

deviator stress, $(\Delta \sigma_d)_f = 63.7\ kN/m^2$

pore pressure, $(\Delta u_d)_f = 47.6\ kN/m^2$

Calculate the undrained friction angle and the drained friction angle of the clay.

NCKU 1032 Qualify Exam for Ph.D. Candidate
Course: Engineering Geology
Time limitation: 100 min.

Note: Make rational assumptions if necessary

Question 1: briefly explain the following terms: (40 pts)

(a) Dip slope
(b) Metamorphism
(c) Weathering
(d) Lugeon test
(e) Primary weak planes

Question 2:

List three well known and widely used techniques for measuring the in situ stresses in rock and briefly describe the procedures of these techniques. (30 pts)

Question 3: questions related to graphical presentation of geological data: (30 pts)

(a) Express the dip direction/dip of a plane with attitude of N30°W/60°N (strike/dip). (10 pts)
(b) List the four main types of rock slope failure and the corresponding stereographic projection plots of structural conditions to give rise to these failures.(20 pts)

103 學年度第 2 學期博士學位候選人資格考試

高等瀝青材料學 (20% for each question)

1. Translate the following paragraph.
   The performance of a pavement is affected by the type, time of application, and quality of the maintenance it receives. Preventive timely maintenance slows the rate of pavement deterioration due to traffic and environmentally applied loads. Delays in maintenance and deferred maintenance increase the quantity of defects and their severity so that when corrected, the cost of repair is greater. Continued deferral of maintenance and rehabilitation actions shortens the time between overlays and reconstruction, and thus increases the life cycle costs of a pavement considerably.

2. Briefly describe the asphalt cement grading system.

3. Briefly describe the reasons to develop the superpave asphalt binder tests and specifications.

4. Briefly describe the procedures of the superpave mix design method.

5. Briefly describe the two principal types of HMA facilities (mixing plants).

103學年度第二學期博士學位候選人資格考試

工程統計 Engineering Statistics

作答方式：Open Book　考試時間：100分鐘　及格分數：60分

1. There are three ways of transporting material from Taipei to Kaohsiung, namely, by highway, rail, or air. Half of the materials are transported by highway, 40% by rail, and the rest by air. The percentages of damaged cargo are 8% by highway, 5% by rail, and 3% by air, respectively.
   (a) What percentage of all cargos may be expected to be damaged? (10%)
   (b) If a damaged cargo is received, what is the probability that it was shipped by rail? (10%)

2. Let $X_1, X_2, \ldots, X_n$ be a random sample from the exponential distribution with parameter λ in the following:

$$f_X(x) = \lambda e^{-\lambda x}, \quad x \geq 0$$

   (a) Find the moment estimator of λ. Is this an unbiased estimator? And why? (15%)
   (b) Find the maximum likelihood estimator of λ. (10%)

3. A sample of 12 readings are as follows:

   105.6, 90.9, 91.2, 96.9, 96.5, 91.3, 100.1, 105.0, 99.6, 107.7, 103.3, 92.4

   Does this data suggest that the population mean reading differs from 100? State and test the appropriate hypotheses using $\alpha = 5\%$. (30%)

4. An empirical relation between variables X and Y is proposed as follows:

$$Y = aX^b$$

   The observed data is given by $(x_i, y_i), i = 1, 2, \ldots, n$.
   (a) Give the formula for the sum of squared errors in order to perform the **linear** regression of $\ln Y$ on $\ln X$ to estimate coefficients a and b. (15%)
   (b) Give the formula to estimate the conditional variance $Var(\ln Y | x)$. (10%)