Examination for Elasticity

Department of Civil Engineering National Cheng Kung University

*Return question-paper after exam
*Closed-book exam

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1. (20 points) Given the relations

$$\sigma_{ij} = s_{ij} + \frac{1}{3}\sigma_{kk}\delta_{ij}, \quad J_2 = \frac{1}{2}s_{ij}s_{ji}, \quad J_3 = \frac{1}{3}s_{ij}s_{jk}s_{ki},$$

where σ and s are symmetric second-order tensors. Show that

(a) $s_{ii} = 0$.

(b) $J_{2,\sigma ij} = s_{ij}$.

(c) $J_{3,\sigma ij} = s_{ik}s_{kj} - 2J_2\delta_{ij}/3$.

(d) For anisotropic fiber-reinforced composites, the joint invariants

$$I = d_i d_j s_{jk} s_{ki}, \quad I_0 = d_i d_j s_{ij},$$

are frequently used in modeling the yield criterion, where d is the unit vector and $d_i d_j$ the directional dyad. Derive $I_{,\sigma ij}$ and $I_{0,\sigma ij}$.

2. (20 points) For isotropic linear elastic materials, prove the following relations between the elastic moduli E (Young's modulus), G (shear modulus), ν (Poisson's ratio) and k (bulk modulus):

$$\nu = \frac{3k - E}{6k}, \quad k = \frac{GE}{9G - 3E}.$$

Using the fact that for an isotropic body the principal axes of stress and strain coincide and assuming that the stress-strain relation is linear so that superposition holds, derive

$$\varepsilon_{ij} = \frac{1+\nu}{E}\sigma_{ij} - \frac{\nu}{E}\sigma_{mm}\delta_{ij},$$

directly from the definitions of E and ν given by

$$E = \frac{\sigma_{11}}{\varepsilon_{11}}, \quad \nu = -\frac{\varepsilon_{22}}{\varepsilon_{11}} = -\frac{\varepsilon_{33}}{\varepsilon_{11}},$$

in simple tension test.

3. **(30 points)** The elastic complementary energy density U^c is defined viz.

$$U^c \equiv \int_0^{\sigma_{ij}} \varepsilon_{ij} d\sigma_{ij}.$$

Based on this concept, the behavior of an isotropic nonlinear elastic material is described by the following assumed polynomial expression for U^c:

$$U^c(I_1, J_2, J_3) = aI_1^2 + bJ_2 + cJ_3^2,$$

where a, b and c are constants and I_1, J_2 and J_3 are the stress invariants defined in Problem 1 of this exam.

(a) Derive the stress-strain relations for this material in terms of the constants a, b and c.

(b) Derive the stress-strain relations in simple tension. Find an expression for the tangent Young's modulus E_t defined viz.

$$E_t \equiv \frac{d\sigma}{d\varepsilon},$$

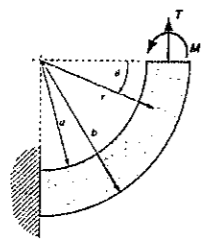
in simple tension, in terms of the stress σ. What is the value of the initial tangent Young's modulus $E_t(0)$ (at $\sigma = 0$)?

(c) Show that the constitutive equations are reduced to those of the isotropic linear elastic material for $c = 0$. Find the relations between the constants a, b, and the elastic moduli E, ν for this case.

4. **(20 points)** Show that the curved beam problem with end loadings shown in the figure can be solved by superimposing the solution of the Airy stress function

$$\phi = \left(Ar^3 + \frac{B}{r} + Cr + Dr\log r\right)\cos\theta$$

with that from the pure bending problem.
What are the stress and displacement fields?



5. **(10 points)** For a problem of plane stress,

$$Eu = (1-\nu)(\log r)\cos\theta - 2\cos\theta + 2\theta\sin\theta$$
$$Ev = (1-\nu)(1-\log r)\sin\theta + 2\theta\cos\theta$$

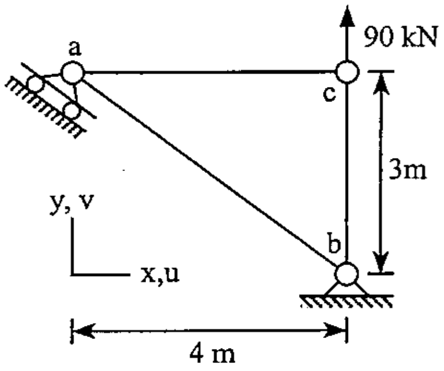
where (u,v) are the displacement components in polar coordinates (r,θ), E is the elasticity modulus, and ν is the Poisson's ratio. There is no body force.

(a) Is this a possible displacement vector of the origin is included in the body? Why?
(b) Is this a possible displacement vector for a closed ring with center at the origin? Explain.
(c) Does the corresponding Airy stress function satisfy the biharmonic equation? Explain.
(d) Show that for this problem the in-plane stress components are equal.

Finite Element Method

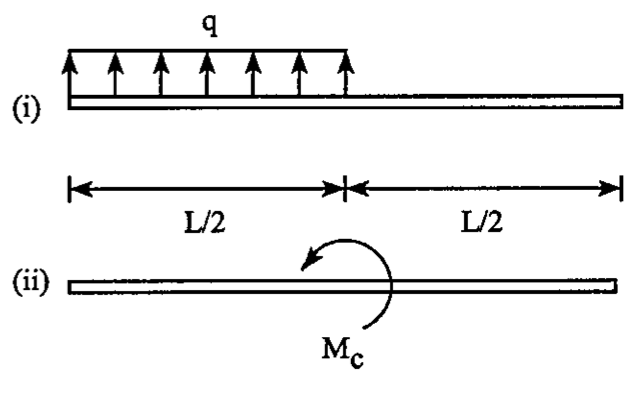
(Close book, 100 minutes, 70% to pass)

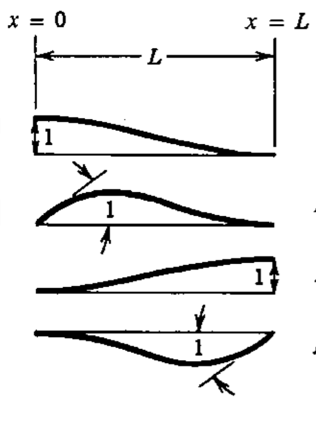
1. The structural stiffness equation of the truss shown is $[K]\{D\}=\{R\}$, where $\{D\}=\{u_a, v_a, u_b, v_b, u_c, v_c\}^T$ and $\{R\}=\{P_{xa}, P_{ya}, P_{xb}, P_{yb}, P_{xc}, P_{yc}\}^T$. Assume E = 200 GPa and the cross section area $A = 1000\ mm^2$ for all bars. The truss is subjected to a settlement $v_b = -20$ mm. (i) Obtain the structural stiffness matrix [K] of the truss. (ii) Calculate the displacements u_a, v_a, u_c, v_c. (iii) Calculation the reactions $P_{xa}, P_{ya}, P_{xb}, P_{yb}$ at nodes a and b. (20%)



2. (a) What is the parasitic shear? Use the four-node isoparametric plane element with full integration rule to illustrate it. (8%)
   (b) What is the zero-energy mode (hourglass mode)? Use the four-node isoparametric plane element with reduced integration rule to illustrate it. (7%)

3. Determine the consistent nodal load vector $\{r_e\}_{4\times1}$ of a beam element with length L for the following two loads: (i) Uniformly distributed transverse load q acts on the left half of the element. (10%) (ii) Concentrated moment M_c applies at midspan. (10%)





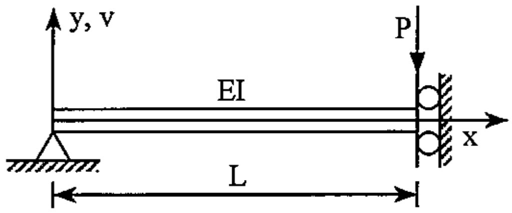
$$N_1 = 1 - \frac{3x^2}{L^2} + \frac{2x^3}{L^3}$$

$$N_2 = x - \frac{2x^2}{L} + \frac{x^3}{L^2}$$

$$N_3 = \frac{3x^2}{L^2} - \frac{2x^3}{L^3}$$

$$N_4 = -\frac{x^2}{L} + \frac{x^3}{L^2}$$

4. Consider a beam of uniform EI that supports a concentrated load P as shown. Assume the lateral deflection of the beam is v(x). (i) Calculate the potential energy Π_p of the beam. (ii) Use the calculus of variation to find the governing differential equation and nonessential boundary conditions of the beam. (15%)



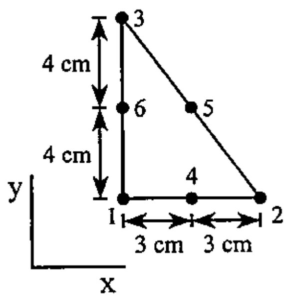
5. (a) Evaluate the following integral by using 3-point Gaussian quadrature. (10%)

$$I = \int_{-2}^{3}\left[x^2 + \cos\left(\frac{x}{2}\right)\right]dx$$

(b) Is the integral evaluation in part (a) an exact? Why or why not? (5%)

Hint: $\xi = -\sqrt{0.6}, 0, \sqrt{0.6}$ and $W = 5/9, 8/9, 5/9$.

6. The triangular element shown below is subjected to the body force $f_x = 10\ \mathrm{N/cm^3}$. Calculate the consistent nodal load vector $\{Q\} = \{Q_{1x}, Q_{2x}, Q_{3x}, Q_{4x}, Q_{5x}, Q_{6x}\}^T$ using the area coordinates. Assume the thickness of the plate is $t = 2$ cm. (15%)



102 學年度第 1 學期土木所博士學位候選人資格考試

結構動力學 (題目卷共2頁)

及格分數: 60 分　考試時間: 100 分鐘

1. For the given earthquake design response spectra as shown in Figs. 1a and 1b,

   (a) Please find the max. relative displacement and the max. absolute acceleration of the given single-degree-of-freedom (SDOF) system as shown in Fig. 2. (10%)

   (b) By adding a small mass and spring on top of the SDOF, please find the max. relative displacements and the max. absolute accelerations of u_1 and u_2 of the 2-DOF system (Fig. 3). Solve this problem by using the square-root-of- sum-of-squares (SRSS) rule. (40%)

   (c) Compare the results and give your comments to explain max. responses of u_1. (10%)

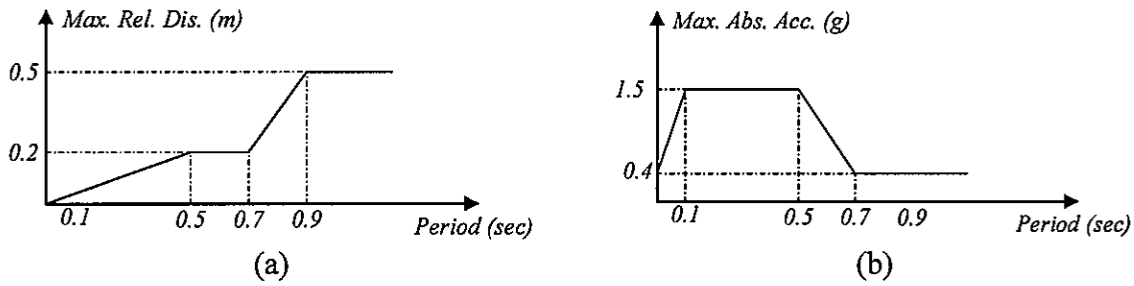


Figure. 1

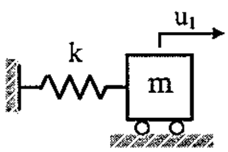


Figure. 2 (Given mass $m = 10\,ton$, spring constant $k = 640\pi^2\,kN/m$)

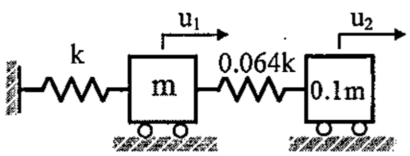


Figure. 3

2. Please derive the complete solution for at-rest initial conditions of an undamped single-degree-of-freedom system subjected to a harmonic force. i.e. $m\ddot{u} + ku = P_0 \sin \omega_n t$. Note: $\omega_n = \sqrt{k/m}$. (20%)

3. Please give a brief description of the following questions (you may define qualitatively the required parameters if not given and also give the definition if any terminology is used) (**<u>NOTE: No difficult calculation is required!</u>**) ：(20%)

(a) What are the procedures to construct the combined D-V-A response spectra of a given ground motion? Please make a sketch and also give brief descriptions regarding the acceleration-sensitive region, the velocity-sensitive region, and the displacement-sensitive region. (10%)

(b) What is the important distinction between earthquake elastic design spectrum and inelastic design spectrum? For ground motion on firm ground, $T_a = 1/33$ sec, $T_b = 1/8$ sec, $T_e = 10$ sec, and $T_f = 33$ sec, assume any given ζ and

$$R_y = \begin{cases} 1 & T_n < T_n \\ \sqrt{2\mu - 1} & T_b < T_n < T_c \\ \mu & T_n > T_c \end{cases}$$

How to construct a constant-ductility design spectrum (for $\mu = 4$) from the elastic design spectrum? (make a sketch if necessary) (10%)

NCKU 1021 Qualify Exam for Ph.D. Candidate
Course: Soil mechanics
Time limitation: 100 min.

Note: There are two pages in the question sheet. Make rational assumptions if necessary

Question 1: briefly explain the following terms: (25 pts)

(a) Overcompaction
(b) Overconsolidation ratio
(c) ϕ=0 concept
(d) Thixotropy
(e) Equipotential line

Question 2: questions related to physical properties of soils; (25 pts)

(a) For a given soil with following properties: $G_s = 2.74$, $\gamma_t = 20.6$ kN/m^3, w=16.6%, plot the three phase diagram and determine the following properties accordingly: (a) dry unit weight, (b) void ratio, and (c) porosity. (7 pts)
(b) List the physical properties required for USCS soil classification and explain why these properties are involved. (6 pts)
(c) Describe the physical meanings of the three lines on Casagrande plasticity chart. (6 pts)
(d) A clay soil was tested in the laboratory and found to have the following properties:

(a) $\gamma = 12.54$ kN/m^3, (b) e=9.0, (c) S=95%, (d) $G_s = 2.75$, (e) ω=311%. One of the values is inconsistent with the others. Find the inconsistent value and report it correctly. Show all your computations and phase diagrams.(6 pts)

Question 3: questions related to shear strength of soils: (25 pts)

(a) The state of stress of a soil element is shown in Fig. 1. Plot the Mohr circle, pole, and principle planes. Additionally, calculate the normal and shear stresses on plane AB. (7 pts)

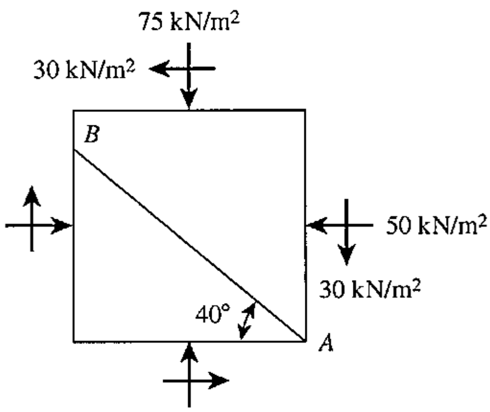


Fig. 1

(b) Define the MIT stress path and express the (m, α) of K_f-line ($q = m + p' \tan\alpha$) in terms of (c', ϕ'). (6 pts)

(c) A SCU test on NC clay yields the following results: effective consolidation pressure, $\sigma_3' = 100$ kPa; deviator stress at failure, $\Delta\sigma_{d,f} = 125$ kPa; and excess pore pressure at failure, $\Delta u_{d,f} = 75$ kPa. Calculate the total and effective strength parameters of the soil. (6 pts)

(d) Plot the stress path of (c) along with the K_f-line. (6 pts)

Question 4: questions related to soil consolidation: (25 pts)

(a) Calculate the settlement of the clay layer in Fig. 2. (9 pts)

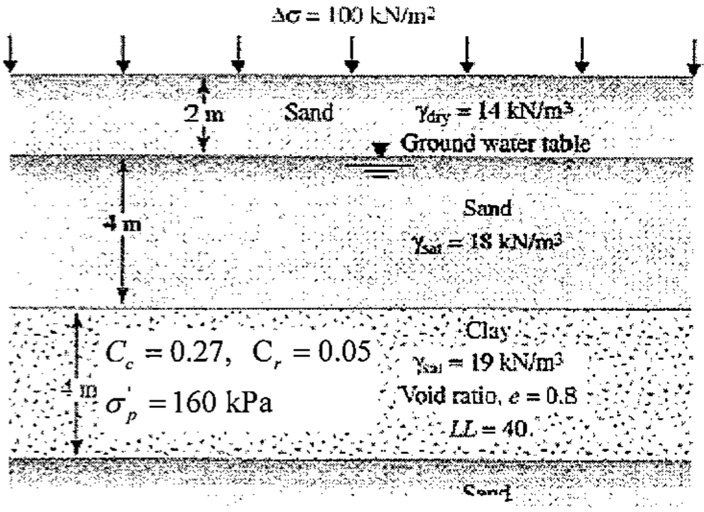


Fig. 2

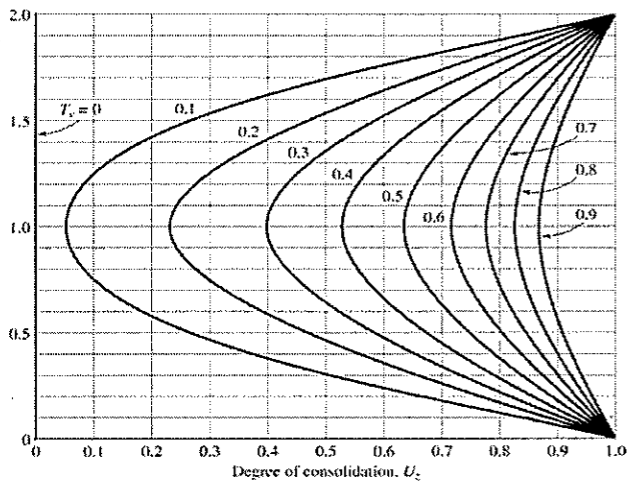


Fig. 3

(b) Figure 3 shows the progress of a consolidation case. State the boundary and initial conditions of Fig. 3. (6 pts)

(c) If the coefficient of consolidation (c_v) of the clay layer in Fig. 2 is $2.4\ \text{m}^2/year$ and the progress of consolidation fits the Fig. 3 conditions, calculate the degree of consolidation and excess pore pressures after 1 year at the depths of 6, 7, 8, 9, and 10 m from surface in Fig. 2. (10 pts)

102 學年度第一學期博士學位候選人資格考試

考試科目: 鋼鐵材料與結構

考試方式: Closed Book
考試時間: 100 分鐘

1. Discuss temperature-induced martensitic phase transformation in carbon steel, and its effects on mechanical properties. **(10%)**

2. According to the iron-carbon binary phase diagram, roughly determine the percentage of all phases in 3.5 weight % carbon steel at 800 °C by the lever rule. **(10%)**

3. Why is cast iron inferior to steel? Please discuss through correlations between mechanical properties and microstructures? **(10%)**

4. How to obtain pearlite microstructure in steel? What are the advantages of having the pearlite microstructure? **(10%)**

5. Alloying steel may alter the mechanical properties of steel. Please discuss the effects of Mo, V, Ni, Cr and Si, respectively, on the mechanical properties of carbon steel. **(10%)**

6. The AISC-LRFD (3rd Edition) provides an approximate method using the following equation (Equation (1)) to consider second-order effects for M_u in beam-columns and frames:

$$M_u = B_1 M_{nt} + B_2 M_{lt} \qquad \text{Equation (1)}$$

(a) Please explain the detailed meanings of M_{nt}, M_{lt}, B_1, B_2 and Equation (1). **(15%)**

(b) Please describe the procedure of using Equation (1) to perform second-order analysis for the frame in Figure (1). **(10%)**

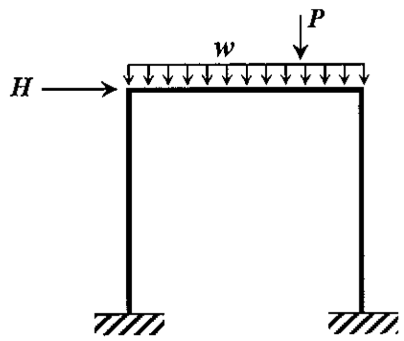


Figure (1)

7. Consider the frame with the boundary conditions of hinge at A and fixed end at D as shown in Figure (2). This frame is loaded with a horizontal force P at E and a vertical force $1.5P$ at F. Member AB and member CD have the same plastic moment of 300 kip-ft (i.e., $M_{p,AB}=M_{p,CD}=300$ kip-ft). Member BC has the plastic moment of 1050 kip-ft (i.e., $M_{p,BC}=1050$ kip-ft). Please determine the maximum value of P for this frame by using plastic analysis. **(25%)**

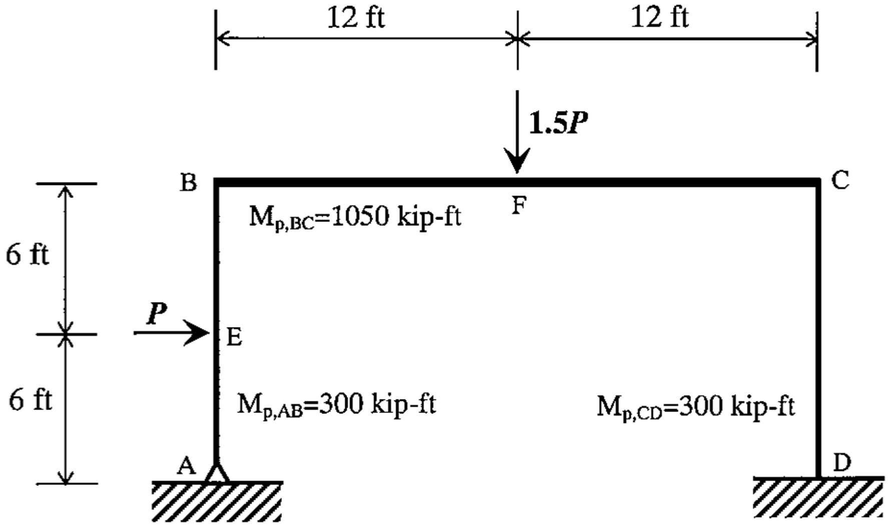


Figure (2)

Finite Element Method

(Close book, 100 minutes, 70% to pass)

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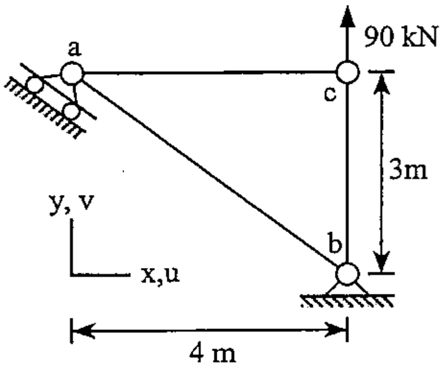

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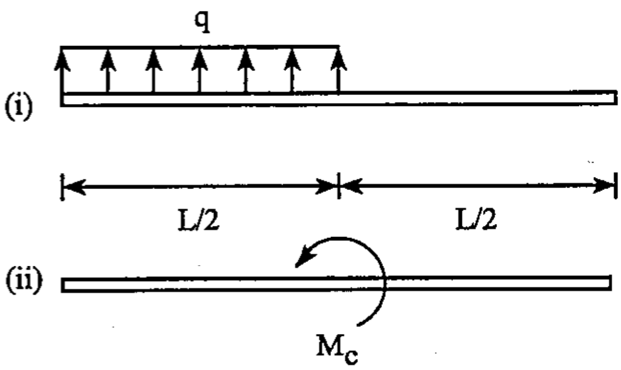

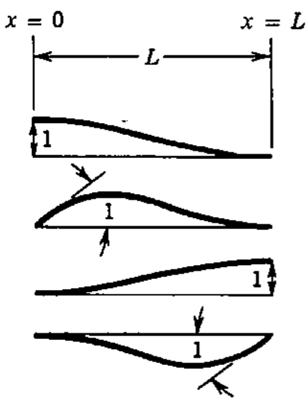

$$N_1 = 1 - \frac{3x^2}{L^2} + \frac{2x^3}{L^3}$$

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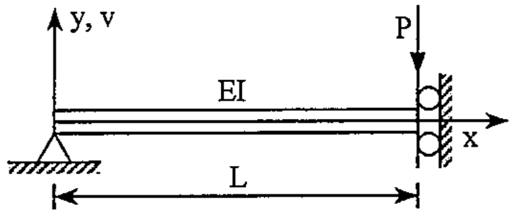

5. (a) Evaluate the following integral by using 3-point Gaussian quadrature. (10%)

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(b) Is the integral evaluation in part (a) an exact? Why or why not? (5%)

Hint: $\xi = -\sqrt{0.6}, 0, \sqrt{0.6}$ and $W = 5/9, 8/9, 5/9$.

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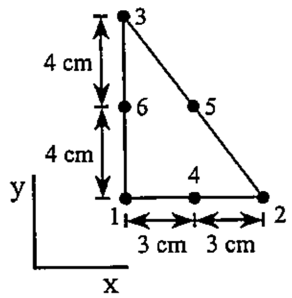

材料機械性質

1. Please answer the following questions pertinent to strength of materials. (20%)
   (a) Derive a relationship between theoretical shear strength and shear modulus, based on your choosing interatomic potential.
   (b) Discuss how dislocations reduce the ideal shear strength of materials.
   (c) Derive a relationship between ideal cohesive strength and Young's modulus, based on your choosing interatomic potential.
   (d) Discuss how voids reduce the theoretical cohesive strength of materials.

2. Please answer the following questions regarding elastic constants of materials. (20%)
   (a) Show that a simple cubic crystal requires three independent elastic constants to model its mechanical behavior in the framework of linear elasticity.
   (b) Estimate the Poisson's ratio of a close-packed single crystal from geometric considerations.
   (c) Show that transversely isotropic materials require five independent elastic constants.
   (d) For isotropic materials, establish the inter-relationships among bulk modulus, shear modulus and Poisson's ratio.

3. Please answer the following questions about plastic behavior of metallic materials. (10%)
   (a) Demonstrate that if a crystal does not have at least five independent slip systems, plastic deformation is not allowed (after von Mises).
   (b) Propose a method to construct three-dimensional yield criteria from experimentally measured one-dimensional yield strength.

4. Suppose a material has the following elastic-plastic deformation behavior for uniaxial stress-strain relation,

   $$\varepsilon = \frac{\sigma}{E} + \left(\frac{\sigma}{H}\right)^{1/n}, \text{ where } E, H, n \text{ are constants}$$

   Estimate the stress-strain curve $\gamma = f(\tau)$ for a state of pure planar shear stress τ, by considering the effective stress $\bar{\sigma}$ (Mises stress) and the effective strain $\bar{\varepsilon}$ follows the same behavior as uniaxial stress and strain. (15%).

5. (a) Explain what K-field is in LEFM, and when it may not exist. (10%)
   (b) If K exists, what parameters does K depend on? Also, explain the use of K. (5%).

6. (a) Explain the life estimate of a cracked component subjected to a cyclic loading. (10%)
   (b) Explain the physical mechanisms of creep in metals, polymers and concrete. (10%)

Please answer in English or Chinese. Answer all the questions concisely.

The traveling salesman problem (TSP) is one of the most heavily studied mathematical problem ever existed. Please answer the following questions.

1. (20%) Please define the TSP.
2. (20%) Why is the TSP so hard to solve?
3. (10%) Suggest a way to solve the TSP with the branch-and-bound method.
4. (10%) Suggest a way to solve the TSP with the Tabu search heuristic.
5. (20%) Please define the longest path problem.
6. (20%) Suppose there is a very efficient algorithm to solve the longest path problem. How can you use this algorithm to solve the TSP?

Qualification CMIS 2013

1. Please identify and explain the rule violations and formating errors within the DFD shown in Figure 1. (10%)

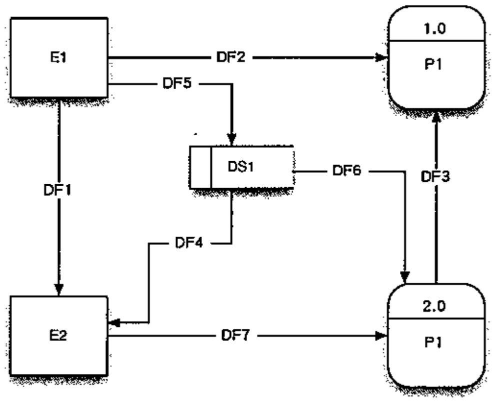

Figure 1

2. What are update anomalies, addition anomalies, and deletion anomalies within the data redundancy? (20%)
3 Transform the E-R diagram of Figure 2 into a set of 3NF relations. (20%)

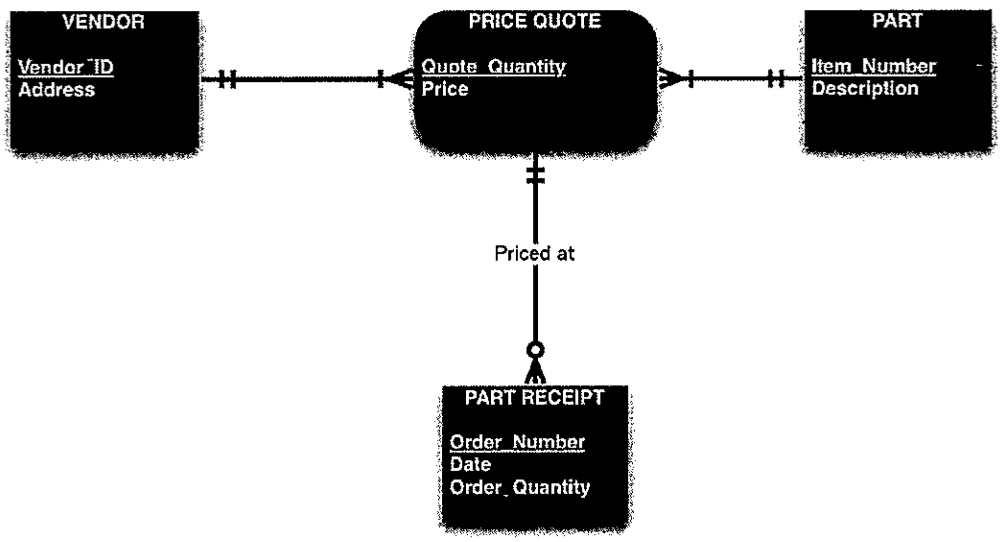

Figure 2

4. What is VPN(Virtual Private Network)? How can we apply VPN to the construction industry? (10%)
5. What is OLAP (Online Analytical Process)? If you were a material supplier how can the OLAP system help you develop marketing strategies? (10%)
6. Use the following business rules to answer the questions:
    a. A department employs many employees, but each employee is employed by one department.
    b. Some employees, know as "Rover," are not assigned to any department.
    c. A division operates many departments, but each department is operated

by one division.

d. An employee may be assigned to many projects and a project may have many employees assigned to it.
e. A project must have at least one employee assigned to it.
f. One of the employees manages each department.
g. One of the employees runs each division.

6.1. Develop an appreciate E-R diagram (10%)
6.2. Write all the cardinalities into the model.(10%)
6.3. Modify the E-R model by splitting the M:N relationship into two 1:M relationships that are connected through a composite entity. Then rewrite the connectivities and cardinalities to match the changes you have made. (10%)