Qualifying examination (Elasticity)

1. Given the stress components

$$\sigma_{11} = k(x^2 y - \frac{2}{3} y^3)$$

$$\sigma_{22} = \frac{1}{3} k y^3$$

$$\sigma_{12} = -kxy^2$$

$$(k = cons\tan t)$$

(a) Show that they satisfy the equilibrium and compatibility conditions for no body forces, and find the corresponding Airy stress function. (15%)

(b) Show that the Airy stress function is biharmonic.(10%)

2. A rectangular dam is subjected to pressure $p(x_2) = \rho_w x_2$ on one face, where ρ_w is the weight density of water. The dam is made from concrete, with weight density ρ_c (and is therefore subjected to a body force $\rho_c e_2$ per unit volume).

(a) Write down the boundary conditions on all four sides of the dam.(10%)

(b) Consider the following approximate state of stress in the dam

$$\sigma_{22} = \frac{\rho_w x_2^3 x_1}{4a^3} + \frac{\rho_w x_2 x_1}{20a^3}\left(-10x_1^2 + 6a^2\right) - \rho_c x_2$$

$$\sigma_{11} = -\frac{\rho_w x_2}{2} + \frac{\rho_w x_2 x_1}{4a^3}\left(x_1^2 - 3a^2\right)$$

$$\sigma_{12} = \frac{3\rho_w x_2^2}{8a^3}(a^2 - x_1^2) - \frac{\rho_w}{8a^3}\left(a^4 - x_1^4\right) + \frac{3\rho_w}{20a}(a^2 - x_1^2)$$

Show that (i) The stress state satisfies the equilibrium equations (5%) (ii) the stress state exactly satisfies boundary conditions on the sides $x_1 = \pm a$,(5%) (iii) The stress does not satisfy the boundary condition on $x_2 = 0$ exactly. (5%)

(c) Show, however, that the resultant *force* acting on $x_2 = 0$ is zero, so by Saint Venant's principle the stress state will be accurate away from the top of the dam. (5%)
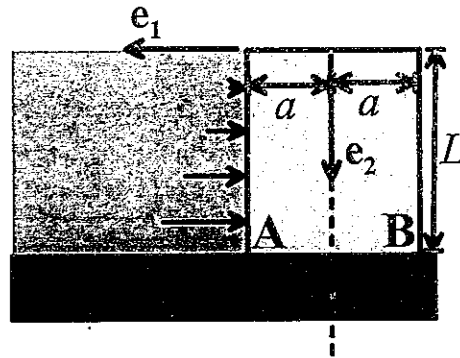
(3) A rigid circular inclusion of radius a is embedded in an infinitely extended two-dimensional medium where uniform stress p is applied at infinity as shown in Fig. The solution to this problem may be formulized as follows.(conditions of plane-strain are assumed) (a) First show that the substitution of the strain-displacement expressions

$$\varepsilon_{rr} = \frac{du}{dr}, \qquad \varepsilon_{\theta\theta} = \frac{u}{r}$$

into the stress-strain relations

$$\sigma_{rr} = 2\mu\varepsilon_{rr} + \lambda(\varepsilon_{rr} + \varepsilon_{\theta\theta}),$$
$$\sigma_{\theta\theta} = 2\mu\varepsilon_{\theta\theta} + \lambda(\varepsilon_{rr} + \varepsilon_{\theta\theta}),$$

will give

$$\sigma_{rr} = (\lambda + 2\mu)\frac{du}{dr} + \lambda\frac{u}{r},$$

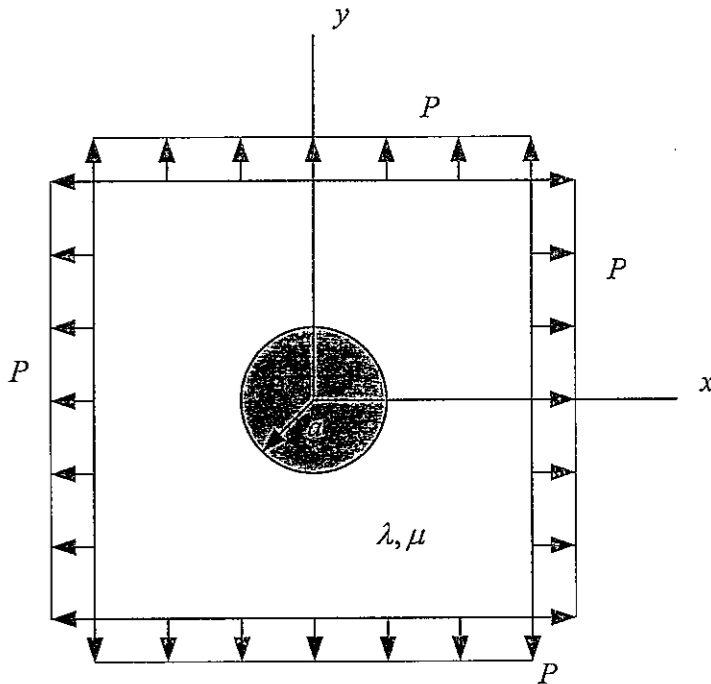$$\sigma_{\theta\theta} = \lambda\frac{du}{dr} + (\lambda + 2\mu)\frac{u}{r}, \quad (5\%)$$

(b) Obtain an equation for u when these stresses are introduced into the equilibrium equation

$$\frac{d\sigma_{rr}}{dr} + \frac{1}{r}(\sigma_{rr} - \sigma_{\theta\theta}) = 0, \quad (5\%)$$

(c) Show that the solution for the equation of u is $u = Ar + \dfrac{B}{r}$ $\quad$ (5%)

where A and B are arbitrary constants.

(d) Find the stress components of the medium. (10%)

4 The displacement field in a 2D elastic body is given by

$$u_1 = q[5x_1 + (x_2 - 2)^2]$$
$$u_2 = q[4x_2 + (x_1 - 1)^2]$$

where q is a small positive constant.

(i) Determine the strain tensor ε_{ij} and the rotation tensor ω_{ij} (10%).

(ii) At the point $r = (3,2)$ find the extension ε_n of the line element $n\,ds$, where $n = (3/5,\ 4/5)$. (10%)

Dynamics of Structure

1. Please give a brief description of the following questions (you may define qualitatively the required parameters if not given and also give the definition if any terminology is used) **(NOTE: No difficult calculation is required!)** ：(30%)

    (a) What are the procedures to construct the combined D-V-A response spectra of a given ground motion? Please make a sketch and also give brief descriptions regarding the acceleration-sensitive region, the velocity-sensitive region, and the displacement-sensitive region. (15%)

    (b) What is the important distinction between earthquake elastic design spectrum and inelastic design spectrum? For ground motion on firm ground, $T_a = 1/33$ sec, $T_b = 1/8$ sec, $T_e = 10$ sec, and $T_f = 33$ sec, assume any given ζ and

    $$R_y = \begin{cases} 1 & T_n < T_n \\ \sqrt{2\mu - 1} & T_b < T_n < T_c \\ \mu & T_n > T_c \end{cases}$$

    How to construct a constant-ductility design spectrum (for $\mu = 2, 4, and\ 8$) from the elastic design spectrum? (make a sketch if necessary) (15%)

2. Please find the max. displacements of u_1 and u_2 of the system (Fig. 2a) under the earthquake design response spectrum as shown in Fig. 2b. Solve this problem by using the square-root-of- sum-of-squares (SRSS) rule. (50%)
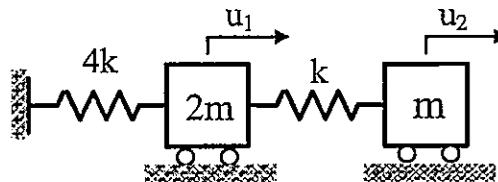

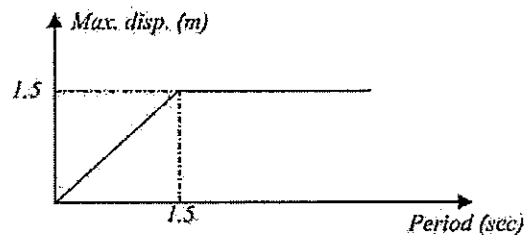Fig. 2a (Given mass $m = 10\ ton$, spring constant $k = 200\ kN/m$)

Fig. 2b

3. Please derive the complete solution for at-rest initial conditions of an undamped single-degree-of-freedom system subjected to a harmonic force. i.e. $m\ddot{u} + ku = P_0 \sin \omega_n t$. Note: $\omega_n = \sqrt{k/m}$. (20%)

# Finite Element Method
(Close book, 100 minutes, 70% to pass)

1. The following True-False questions refer to finite elements based on assumed displacements. (15%)

   ( ) A. The calculated displacements are more accurate than the calculated stresses.

   ( ) B. The diagonal coefficients of a structural stiffness matrix [K] must be greater than or equal to zero.

   ( ) C. The elements should not be able to display rigid body motion and constant strain states.

   ( ) D. First-order (linear) elements with full integration are prone to locking and normally should not be used.

   ( ) E. The 8-node plane elements are less sensitive to the distortion of geometry than the 9-node plane elements.

2. A tapered bar element with three degrees of freedoms $\{d_1, d_2, d_3\}$ and subjected to a concentrated force P is shown below. Assume the Young's modulus of the element is E. If node 2 is condensed out from the element, determine the condensed stiffness matrix $[k]^*_{2\times2}$ and the consistent nodal forces r_1 and r_3 for the element so that $[k]^*_{2\times2}\{d_1, d_3\}^T = \{r_1, r_3\}^T$. (15 Points)
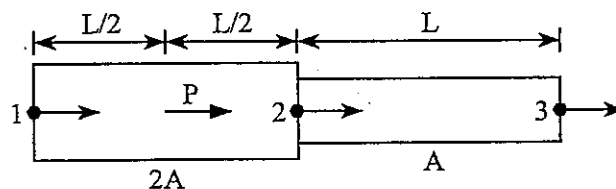
3. Consider a beam of uniform EI that supports a concentrated force P and a concentrated moment M_0 as shown. Assume the lateral deflection of the beam is v(x). (i) Calculate the potential energy Π_p of the beam. (ii) Use the calculus of variation to find the governing differential equation and nonessential boundary conditions of the beam. (20%)
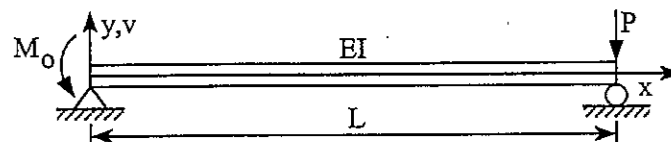
4. (i) For a 4-node plane element with reduced integration rule, how many zero energy modes may exist? Sketch any one of the zero energy mode shape for it and show all the positions of Gauss points. (ii) For a 9-node plane element with reduced integration rule, how many zero energy modes may exist? Sketch any one of the zero energy mode shape for it and show all the positions of Gauss points. (iii) For a 4-node plane element with reduced integration rule, use the sketch you draw to explain why the strain energy at the reduced integration point is zero? (15%)

4-node element   9-node element
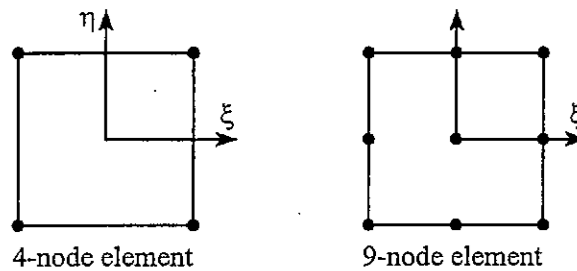
5. (i) Use a 2-point Gauss integration rule to evaluate the following integral: (10%)

$$I = \int_{-1}^{4} \frac{e^{x} \sin x}{x^2 + 1} dx, \qquad (-1 \le x \le 4)$$

(ii) Is the integral evaluation in part (i) an exact? Why or why not? (5%)

Hint: $\xi = \pm 1/\sqrt{3}$.

6. A 4-node isoparametric axisymmetric element is shown below. Let $\{\varepsilon\}^T = \{\varepsilon_r, \varepsilon_\theta, \varepsilon_z, \gamma_{rz}\}^T$ and $\{d\}^T = \{u_1, v_1, u_2, v_2, u_3, v_3, u_4, v_4\}^T$. If the strain-displacement relation of the element is $\{\varepsilon\} = [H][T][Q]\{d\} = [B]\{d\}$, find the explicit expressions for the $[H]_{4\times5}$, $[T]_{5\times5}$, and $[Q]_{8\times8}$ matrices. (20%)

Note: $\{\varepsilon\}^T = [H]\{u_{,r}, u_{,z}, w_{,r}, w_{,z}, u\}^T$, $\{u_{,r}, u_{,z}, w_{,r}, w_{,z}, u\}^T = [T]\{u_{,\xi}, u_{,\eta}, w_{,\xi}, w_{,\eta}, u\}^T$, $\{u_{,\xi}, u_{,\eta}, w_{,\xi}, w_{,\eta}, u\}^T = [Q]\{d\}^T$.

Advanced Bituminous Materials
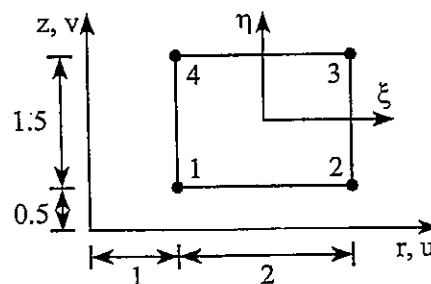
高等瀝青材料學 (20% for each question)

1. Translate the following paragraph.

   Empirical tests are used less frequently now because they are not able to predict performance for conditions that are different than those under which the tests were developed. These tests were never good at predicting performance and have become less reliable in prediction performance as axle loads and tire pressures have continued to increase.

2. Briefly describe the asphalt refining procedures.

3. Briefly describe the major superpave asphalt binder testing equipment and purpose.

4. Briefly describe the procedures of the Marshall mix design and superpave mix design method.

5. Briefly describe the properties of an ideal pavement binder.

1. A random variable X has a normal distribution with mean μ and standard deviation σ.

   (a) Prove $E(X^n) = \mu E(X^{n-1}) + (n-1)\sigma^2 E(X^{n-2})$, where E denotes the expectation. (10%)

   (b) Find $E[(X-\mu)^3]$ and $E(X^3)$. (10%)

2. Determine the maximum likelihood estimators of the two parameters in a lognormal distribution when the sample simple is n. (20%)

3. Given a sample of $n = 10$ as follows:

   $$2.0, \quad 1.3, \quad 6.0, \quad 1.9, \quad 5.1, \quad 0.4, \quad 5.3, \quad 0.7, \quad 4.8, \quad 9.0$$

   (a) Obtain a 95% confidence interval for the mean value. (15%)

   (b) The null and alternative hypotheses are

   $$\begin{cases} H_0 : \mu = 2.5 \\ H_1 : \mu > 2.5 \end{cases}$$

   Test the hypothesis at the 5% significance level. (15%)

   (c) Give your assumptions in answering (a) and (b). (5%)

4. Given the data as follows:

x	1.4	1.5	2.0	2.5
y	23.0	24.5	25.0	29.0

   Calculate the least-square estimators of the slope and y-intercept, and then the coefficient of determination. Assumptions and procedure must be given to obtain the results. (25%)

101 學年度第二學期博士學位候選人資格考試
考試科目: 鋼鐵材料與結構

考試方式: <u>Closed Book</u>
考試時間: <u>100 分鐘</u>

1. Discuss the advantages and disadvantages of the martensitic and austenitic steel, respectively, as a structural material. (10%)

2. Draw the iron-carbon binary phase diagram, and identify the eutectic and eutectoid phase transformation points. Use the lever rule to determine the percentage of all phases in 3 weight % carbon steel at 1000°C. (10%)

3. Why stainless steel is stainless? In other words, why does it have superior properties against corrosion, as compared to standard steel. (10%)

4. Does steel show Bauschinger's effects? How strong is it when compared with other metallic materials? What are the consequences of Bauschinger's effects to the steel materials in terms of energy dissipation? (10%)

5. There are many atomic species can be found in construction steel. In addition to carbon and iron, please list 5 other atomic species and discuss their effects to the mechanical properties of the steel. (10%)

6. Consider the beam-column with fixed-end boundary condition at the both ends, and this beam-column is subjected to a concentrated lateral load Q, a uniformly distributed load w and a constant axial compressive load P as shown in Figure 1. Assume that the material of this beam-column is linearly-elastic. Please derive the end moment (M_A or M_B) equation for this beam-column. (25%)
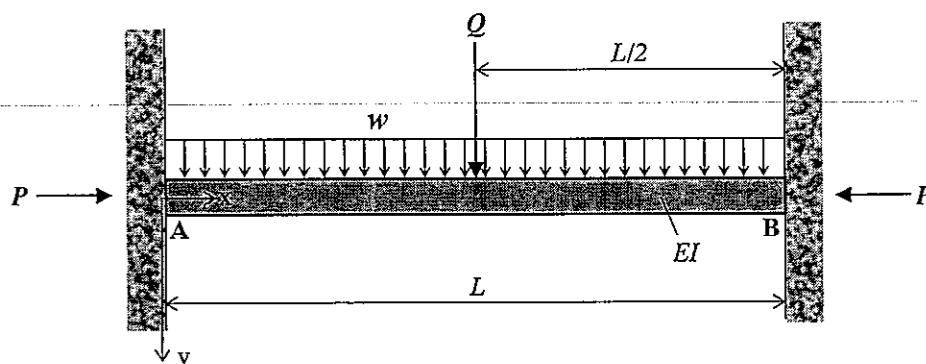

Figure 1

7. Please derive the elastic buckling load P_{cr} for the column with one end pinned at the bottom and one end laterally restrained by a linear spring (spring constant=k) at the top as shown in Figure 2.   Assume that the material of this column is linearly-elastic. (25%)
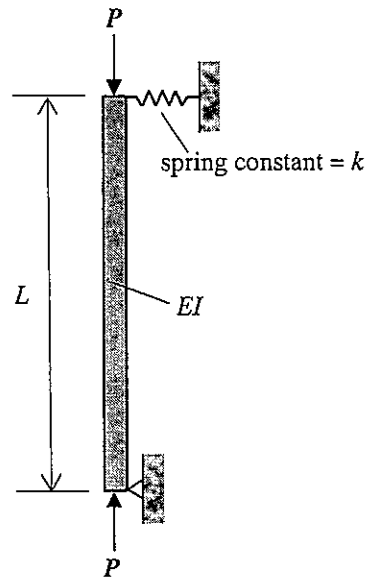

Figure 2

1. Please briefly explain the following terminologies. (14%)

 (a) bcc, fcc and hcp lattice, (b) Bragg's law for determining interatomic spacing, (c) unit cell and Brillouin zone, (d) structural factor, (e) lattice and reciprocal lattice, (f) Wigner-Seitz cell construction, (g) Miller index for crystallographic planes.

2. Please answer the following questions regarding interatomic potential and elastic constants of materials. (18%)

 (a) For the 6-12 Lennard-Jones potential with two potential parameters A and B, please show the potential is stable around the equilibrium point of the two atoms bonded by the potential.

 (b) Calculate the Young's modulus of a simple cubic material when the uniaxial loading is parallel to one of its material axis.

 (c) Calculate all non-zero C_{ijkl} for simple cubic lattice (assume the 1 and 2 axis have different elastic properties) with spring constants k_1 (horizontal interaction between nearest-neighboring atoms), k_2 (vertical interaction between nearest-neighboring atoms) and k_3 (diagonal interaction between diagonally nearest-neighboring atoms on the same atomic planes). Also, please calculate the directional Young's modulus of the simple cubic lattice under uniaxial loading along the [111] crystallographic direction.

3. Please answer the following questions about dislocation and the strength of materials. (18%)

 (a) Please estimate the theoretical (ideal) shear AND cohesive strength of a crystalline solid in terms of their elastic constants.

 (b) Derive Orowan dislocation mobility equation.

 (c) Derive the Hall-Petch relationship to describe the increase of strength with decreasing grain sizes.

4. Derive an equation of pure bending moment as a function of edge strain (strain on top or bottom surface) for a beam with a rectangular cross-section,

(a) for a material with a simple power-hardening stress-strain curve. (10%)

(b) for a material with an elastic and perfectly plastic stress-strain curve. (10%)

5. (a) An infinitely large sheet is subjected to a tensile stress of 350 MPa. There is a central crack of 5 cm long and the material has a yield strength of 500 MPa. Calculate the stress intensity factor and plastic zone size at the crack tip. Also, comment upon the validity of the calculated plastic zone size. (10%)

(b) You have been asked to determine fracture toughness for a material that is supplied in the form of strip. Explain your method of measurement. (10%)

6. Describe and plot the creep behavior of Maxwell material and Vogit material, respectively. (10%)

1. Answer the following questions (20%)
   1.1. What is an example of a procurement activity that might be a valuable inclusion in a network?
   1.2. Conversely, what is an example of a procurement activity that has little merit for inclusion in a network?
   1.3. If "order tiles" requires a lead time of 35 days and "install tiles" can start right after "get surface ready for titling" is finished, will you define the relationship between "order tiles" and "install tiles" as FS/35 ? Why? if not, what relationship would you define?

2. Please finish the following questions according the information listed in Table 1.
   2.1. Develop a Precedence Diagram (Activity on Node). (5%)
   2.2. Determine the ES, EF, LS, LF, TF, FF and full them in the Table 1. (10%)
   2.3. Identify the critical path (5%)
   2.4. Develop an Arrow Activity (Activity on Arrow) and explain what type of dummies if there are any. (5%)
      * Use beginning of day convention. All activity relationships are (FS,0)

Table 1

Activity ID	Duration	Predecessors	ES	EF	LS	LF	TF	FF
1	6	--						
2	9	1						
3	5	1						
4	5	1						
5	8	2						
6	7	2						
7	4	5						
8	15	4, 6						
9	5	7						
10	6	4, 5, 6						
11	12	3, 7						
12	12	8, 10						
13	4	9, 11, 12						

3. The task of forming beams and slabs of one floor requires 20 days with a standard deviation of 2 days.(20%)
   3.1. What duration should be used if there is to be a 90% confidence that the duration would not be exceeded?
   3.2. What is the probability that the duration will be less than 15 days?
   3.3. What is the probability that the duration will be longer than 22 days?

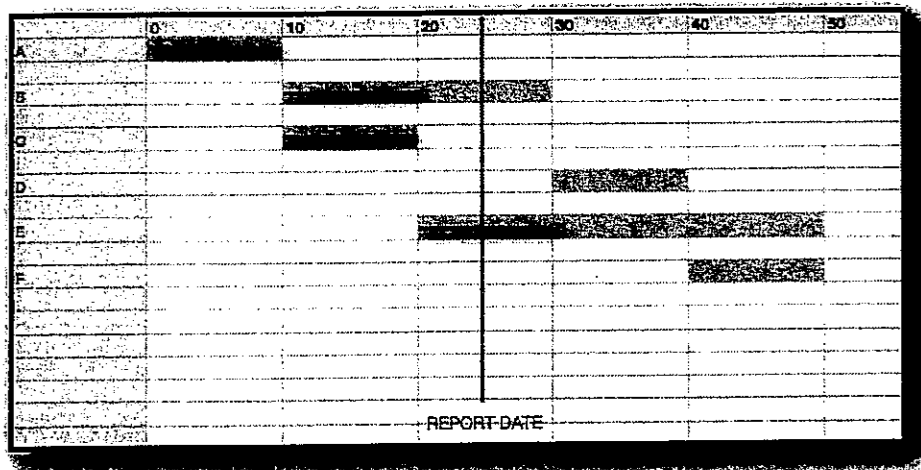4. A construction progress is shown as in Figure 1 and Table 2.

Figure 1

Table 2

Activity	Cost	Scheduled %	Actual %	BCWS	BCWP	ACWP	SPI	CPI	Budget ? Schedule?
A	1000 ?	?	?	?	900 ?	?	?		
B	2100 ?	?	?	?	1100 ?	?	?		
C	3000 ?	?	?	?	3100 ?	?	?		
D	3600 ---	---	---	---	--				
E	2000 ?	?	?	?	750 ?	?	?		
F	8000 ---	---	---	---	--				
Total	19700		?	?	?				
Project SPI	?								
Project CPI	?								
Cost at Completion	?								

4.1. Please fill out the Table 2 to show the project status information of the activities and the construction project. (10%)
4.2. Explain how this construction performs (5%)
4.3. Estimate the cost at completion of this construction project. (5%)

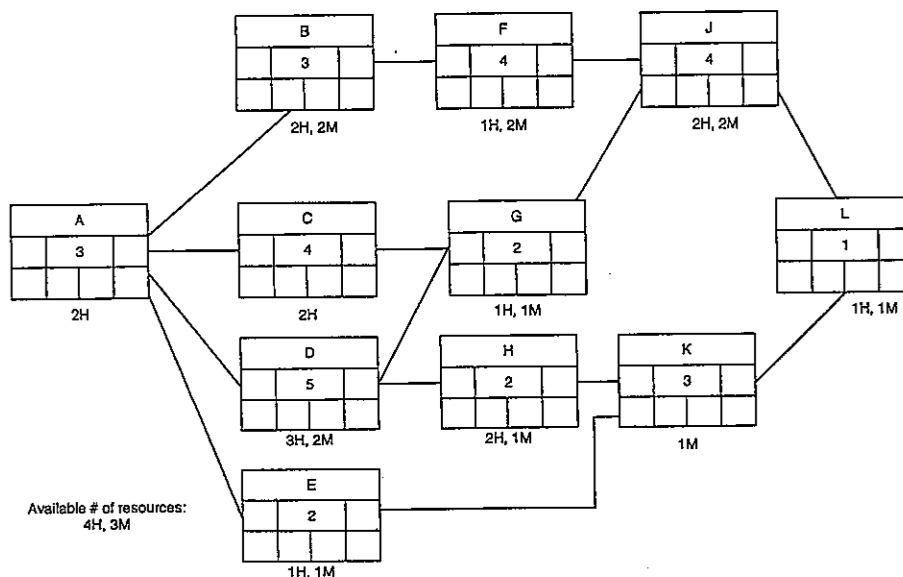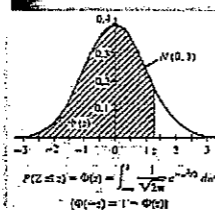5. Please level the resources (R) for the network in Figure 2. (15%)

Figure 2

z	0.00	0.01	0.02	0.03	0.04	0.05	0.06	0.07	0.08	0.09
0.0	0.5000	0.5040	0.5080	0.5120	0.5160	0.5199	0.5239	0.5279	0.5319	0.5359
0.1	0.5398	0.5438	0.5478	0.5517	0.5557	0.5596	0.5636	0.5675	0.5714	0.5753
0.2	0.5793	0.5832	0.5871	0.5910	0.5948	0.5987	0.6026	0.6064	0.6103	0.6141
0.3	0.6179	0.6217	0.6255	0.6293	0.6331	0.6368	0.6406	0.6443	0.6480	0.6517
0.4	0.6554	0.6591	0.6628	0.6664	0.6700	0.6736	0.6772	0.6808	0.6844	0.6879
0.5	0.6915	0.6950	0.6985	0.7019	0.7054	0.7088	0.7123	0.7157	0.7190	0.7224
0.6	0.7257	0.7291	0.7324	0.7357	0.7389	0.7422	0.7454	0.7486	0.7517	0.7549
0.7	0.7580	0.7611	0.7642	0.7673	0.7703	0.7734	0.7764	0.7794	0.7823	0.7852
0.8	0.7881	0.7910	0.7939	0.7967	0.7995	0.8023	0.8051	0.8078	0.8106	0.8133
0.9	0.8159	0.8186	0.8212	0.8238	0.8264	0.8289	0.8315	0.8340	0.8365	0.8389
1.0	0.8413	0.8438	0.8461	0.8485	0.8508	0.8531	0.8554	0.8577	0.8599	0.8621
1.1	0.8643	0.8665	0.8686	0.8708	0.8729	0.8749	0.8770	0.8790	0.8810	0.8830
1.2	0.8849	0.8869	0.8888	0.8907	0.8925	0.8944	0.8962	0.8980	0.8997	0.9015
1.3	0.9032	0.9049	0.9066	0.9082	0.9099	0.9115	0.9131	0.9147	0.9162	0.9177
1.4	0.9192	0.9207	0.9222	0.9236	0.9251	0.9265	0.9279	0.9292	0.9306	0.9319
1.5	0.9332	0.9345	0.9357	0.9370	0.9382	0.9394	0.9406	0.9418	0.9429	0.9441
1.6	0.9452	0.9463	0.9474	0.9484	0.9495	0.9505	0.9515	0.9525	0.9535	0.9545
1.7	0.9554	0.9564	0.9573	0.9582	0.9591	0.9599	0.9608	0.9616	0.9625	0.9633
1.8	0.9641	0.9649	0.9656	0.9664	0.9671	0.9678	0.9686	0.9693	0.9699	0.9706
1.9	0.9713	0.9719	0.9726	0.9732	0.9738	0.9744	0.9750	0.9756	0.9761	0.9767
2.0	0.9772	0.9778	0.9783	0.9788	0.9793	0.9798	0.9803	0.9808	0.9812	0.9817
2.1	0.9821	0.9826	0.9830	0.9834	0.9838	0.9842	0.9846	0.9850	0.9854	0.9857
2.2	0.9861	0.9864	0.9868	0.9871	0.9875	0.9878	0.9881	0.9884	0.9887	0.9890
2.3	0.9893	0.9896	0.9898	0.9901	0.9904	0.9906	0.9909	0.9911	0.9913	0.9916
2.4	0.9918	0.9920	0.9922	0.9925	0.9927	0.9929	0.9931	0.9932	0.9934	0.9936
2.5	0.9938	0.9940	0.9941	0.9943	0.9945	0.9946	0.9948	0.9949	0.9951	0.9952
2.6	0.9953	0.9955	0.9956	0.9957	0.9959	0.9960	0.9961	0.9962	0.9963	0.9964
2.7	0.9965	0.9966	0.9967	0.9968	0.9969	0.9970	0.9971	0.9972	0.9973	0.9974
2.8	0.9974	0.9975	0.9976	0.9977	0.9977	0.9978	0.9979	0.9979	0.9980	0.9981
2.9	0.9981	0.9982	0.9982	0.9983	0.9984	0.9984	0.9985	0.9985	0.9986	0.9986
3.0	0.9987	0.9987	0.9987	0.9988	0.9988	0.9989	0.9989	0.9989	0.9990	0.9990

$$P(Z \leq z) = \Phi(z) = \int_{-\infty}^{z} \frac{1}{\sqrt{2\pi}} e^{-u^2/2} \, du$$

$$[\Phi(-z) = 1 - \Phi(z)]$$