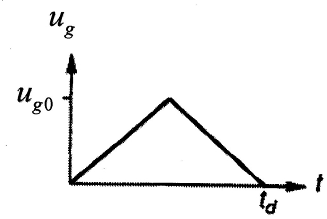
109學年度第一學期成大土木系博士學位候選人資格考試
Qualifying Examination
結構動力學(Structural Dynamics)

及格分數：60分　考試時間：100分鐘

1. For a single-degree-of-freedom system with natural frequency ω_n, damping ratio ζ, and quiescent initial conditions under the **base displacement** shown in the following figure, find the response displacement relative to the base. (20%)



2. For a single-degree-of-freedom system with mass m, damping coefficient c, and stiffness k, find the area under its **impulse response function**, $\int_0^\infty h(t)\,dt$, and explain this results. (20%)

3. The response acceleration time history, $a(t)$, of a single-degree-of-freedom vibrating system under **unknown excitation** is measured. Give a method to evaluate the natural period of vibration of this vibrating system on the basis of $a(t)$. (20%)

4. Under what conditions can a **two**-degree-of-freedom vibrating system be replaced by **two single**-degree-of-freedom vibrating systems for response analysis? (10%)

5. For a three-story shear building, the **frequency response functions** (frequency domain transfer functions) of a **base** acceleration to **floor** absolute accelerations are identified and compared with theoretical ones, as shown partly in the following figures. Point out the floor numbers, 1, 2, or 3, whose frequency response functions are plotted in those two figures respectively, and give your reasons. (20%)

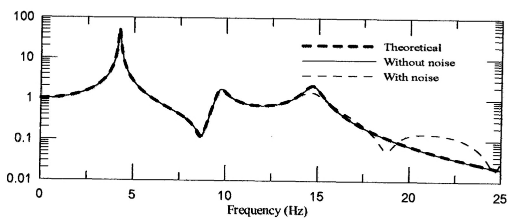


Figure 1

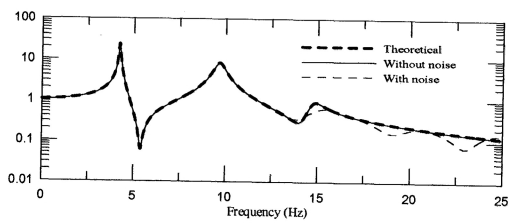


Figure 2

6. A vibrating system with **distributed** mass and elasticity can be modeled as a **multi-degree-of-freedom** system through the finite element method. Are the modal periods increased after such a discretized approach? And why? (10%)

Qualification Examination (109.10)
Finite Element Method

(1) Consider a two-section bar as shown in Fig. 1, in which the left hand side of the bar is clamped, and the right hand side is subjected to a concentrate load P. The strong form of this problem is given as follows:

GE: $$\frac{d}{dx}\left(AE\frac{du(x)}{dx}\right)=0,$$

BCs: $$u(x=0)=0,$$

$$\left[AE\frac{du}{dx}\right]_{x=2L}=P.$$

(a) Find the exact solution. (15%)

(b) Determine the finite element (FE) solutions using two linear elements with uniform spacing. (15%)

(c) Compare the results of displacement and axial force obtained by the analytical and FE approaches. Will the FE solutions be identical to the analytical solutions? State your reasons. (10%)

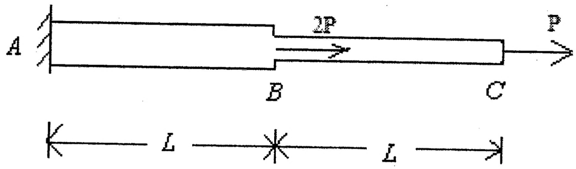


Fig. 1

(2) If nodal values of an element shown in Fig. 2 are $u_i=\hat{u}_i\ (i=1,2,3)$, compute u, $\partial u/\partial x$ and $\partial u/\partial y$ at point $(x,y)=(0.1,0.3)$. (20%)

Hint: The linear interpolation functions $\phi_i^{(e)}(x,y)=\frac{1}{2A_e}\left(\alpha_i^{(e)}+\beta_i^{(e)}x+\gamma_i^{(e)}y\right)\ (i=1,2,3)$ and $\alpha_i^{(e)}=x_j y_k-x_k y_j$, $\beta_i^{(e)}=y_j-y_k$, $\gamma_i=-(x_j-x_k)$, ($i\neq j\neq k$; i,j, and k permute in a natural order).

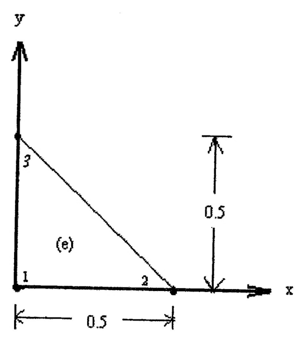


Fig. 2

(continued)

(3) Show that the bilinear interpolation functions for a four-node triangular element in Fig. 3 are of the following form: (20%)

$$\phi_i^{(e)} = A_i + B_i \xi + C_i \eta + D_i \xi \eta \quad (i=1, 2, 3, 4)$$

where $A_1 = 1$, $A_2 = A_3 = A_4 = 0$, $-B_1 = B_2 = 1/a$, $B_3 = B_4 = 0$,

$$C_1 = \frac{6ab - a^2 - 2b^2}{ac(a-2b)}, \quad C_2 = \frac{2b(a+b)}{ac(a-2b)}, \quad C_3 = \frac{a+b}{c(a-2b)}, \quad C_4 = \frac{-9b}{c(a-2b)}$$

$$D_1 = D_2 = D_3 = -\frac{1}{3} D_4 = -\frac{3}{c(a-2b)}.$$

Hint: The coordinate of point 4 is $\left(\frac{a+b}{3}, \frac{c}{3}\right)$.

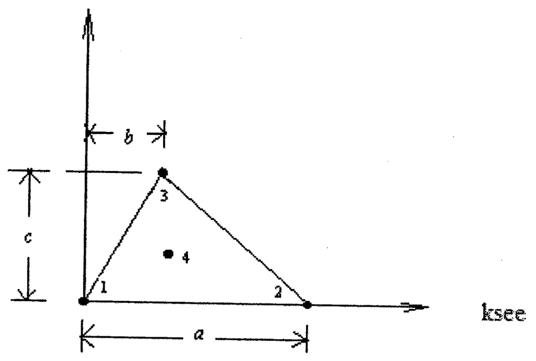


Fig. 3

(4) Give two FE meshes, i.e., a coarse and a fine mesh, for a uniform cantilevel bar under a torque applied at the free end, where the quadratic Serendipity-family elements are used and the cross section of the bar is shown as follows: (20%)
(You need to explain some guidances that you followed to generate your meshes.)

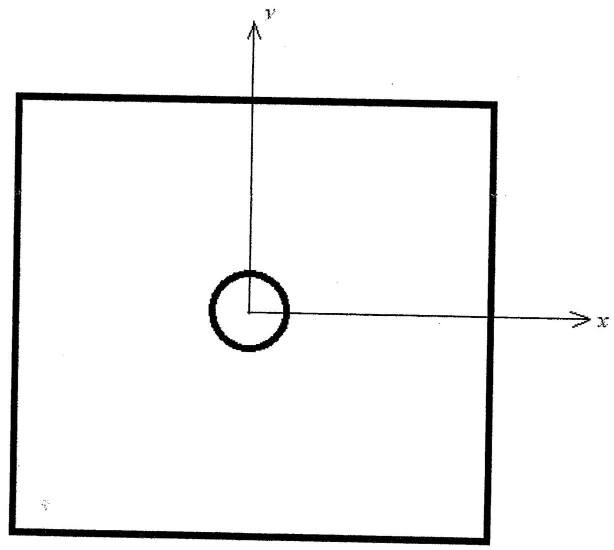


Fig. 4

Engineering Mathematics Qualifying Exam

Problem 1

Solve the following differential equations.

(1) $\dfrac{d^2y}{dx^2}-2\dfrac{dy}{dx}-y=0$ (10%)

(2) $x^2\dfrac{d^2y}{dx^2}-2x\dfrac{dy}{dx}-4y=0$ (10%)

Problem 2

Recall that a square matrix is said to be diagonalizable if there exists an invertible matrix $\mathbf{T}$ so that $\mathbf{T}^{-1}\mathbf{AT}$ is diagonal. Determine whether or not each of the following matrices are diagonalizable. If not diagonalizable, give a reason. If diagonalizable, obtain $\mathbf{T}$.

(1) $\mathbf{A}=\begin{bmatrix}0 & 1 & 0\\ 0 & 0 & 1\\ 0 & 1 & 0\end{bmatrix}$ (10%)

(2) $\mathbf{A}=\begin{bmatrix}0 & 1\\ -1 & 2\end{bmatrix}$ (10%)

Problem 3

Evaluate the surface integral $\iint_S \vec{F}\cdot d\vec{S}$, where $\vec{F}=x\vec{i}+y\vec{j}+z\vec{k}$ and S is the surface defined by $z=xy+1$ over the square $0\le x\le 1$ and $0\le y\le 1$. (20%)

Problem 4

Solve the wave equation $\dfrac{\partial^2 y}{\partial x^2}=\dfrac{1}{c^2}\dfrac{\partial^2 y}{\partial t^2}$, where c is a constant (wave velocity of string), with the boundary conditions $y(0,t)=0$, $y(L,t)=0$, and initial conditions

$y(x,0)=U(x)$, $\partial y(x,0)/\partial t=V(x)$. (20%)

Problem 5

Are the following functions analytic? Explain your answer.

(1) $f(z)=z^2+z$ (5%)

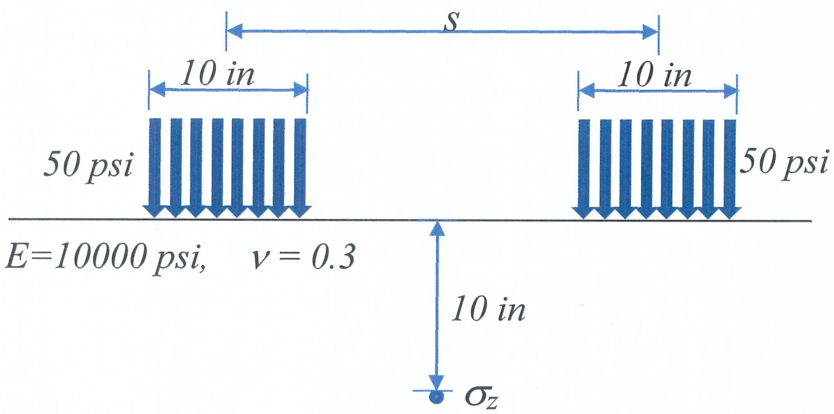
(2) $f(z)=2x^2+y+i(y^2-x)$ (5%)

(3) $f(z)=\dfrac{x}{x^2+y^2}-i\dfrac{y}{x^2+y^2}$ (5%)

(4) $f(z)=e^z$ (5%)

where $z=x+iy$, $i=\sqrt{-1}$.

1. Please plot the curve of wheel spacing (s) versus vertical stress (σ_z) for the pavement shown below **(20%)**



2. Concrete slab A, 15ft long, 12ft wide, 10in thick. Concrete slab B, 36ft long, 12ft wide, 10in thick. Both slabs subjected to a temperature differential of 15 °F. Assuming that k=100pci, and $\alpha=5\times10^{-6}/°F$. Compare the maximum curling stress in the interior and at the edge of the slabs. **(20%)**

3. Given p_t=2.0, D=9in, determine the EALF for a 28kips single axle load and a 36kips tandem-axle load. **(20%)**

4. Please show the typical stress-dependent models of granular material and fine-grained material. **(20%)**

5. Please describe the relationship and difference between Present Serviceability Rating and Present Serviceability Index. **(20%)**

材料機械性質

博士班資格考 October 30, 2020

1. The 9 independent elastic constants of an orthotropic material are given below: $E_{11}=100GPa$, $E_{22}=50GPa$, $E_{33}=25GPa$, $G_{12}=30GPa$, $G_{13}=20GPa$, $G_{23}=10GPa$, $\nu_{12}=0.4$, $\nu_{13}=0.3$, $\nu_{23}=0.2$. Calculate the resulting strains of the orthotropic material when it is subjected to $\sigma_{11}=\sigma_{22}=10MPa$ and $\sigma_{23}=5MPa$. (20%)

2. A metal made from a solid with a yielding strength of $\sigma_y=100MPa$, is under the multiaxial stress state: $\sigma_{11}=\sigma_{22}=\sigma_{33}=\tau_{12}=\sigma$. What is the maximum allowable stress σ before yielding using Tresca yield criterion. (20%)

3. A cracked plate of a material with Young's modulus $E=100GPa$, Poisson's ratio $\nu=0.3$, yield strength $\sigma_{ys}=400MPa$ and mode I fracture toughness $K_{IC}=10MPa\sqrt{m}$, is subjected to a mode I stress intensity factor, $K_I=5MPa\sqrt{m}$. Calculate the mode I plastic zone sizes for plane stress and plane strain when $\theta=90^0$ using von Mises yield criterion.

Note: For LEFM, $\sigma_{xx}=\dfrac{K_I}{\sqrt{2\pi r}}\cos\dfrac{\theta}{2}\left(1-\sin\dfrac{\theta}{2}\sin\dfrac{3\theta}{2}\right)$,

$\sigma_{yy}=\dfrac{K_I}{\sqrt{2\pi r}}\cos\dfrac{\theta}{2}\left(1+\sin\dfrac{\theta}{2}\sin\dfrac{3\theta}{2}\right)$, $\tau_{xy}=\dfrac{K_I}{\sqrt{2\pi r}}\sin\dfrac{\theta}{2}\cos\dfrac{\theta}{2}\cos\dfrac{3\theta}{2}$,

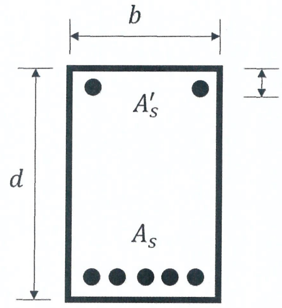
$\sigma_{zz}=\nu(\sigma_{xx}+\sigma_{yy})$ and $\tau_{xz}=\tau_{yz}=0$ (20%)

4. Distinguish between Paris law, Basquin's law and Coffin-Manson law in terms of phenomenology and the mechanism controlling each type of fatigue. (20%)

5. (a) Why does the yield strength of a metal $\sigma_y \propto 1/d$? (b) Why does the creep strain rate of diffusional flow $\dot{\varepsilon}_{ss} \propto 1/d^2$? Note: d is grain size. (20%)

Problem 1. (30%)

A cantilever beam has a design shown below. Beam length = 6000 mm, b=500 mm, d=900 mm, As = 5-D25, and As' = 2-D25. If the required μ_{Δ}=5, find the required μ_{ϕ} and design the appropriate f'c



$\rho = A_s/bd$

$\rho' = A_s'/bd$

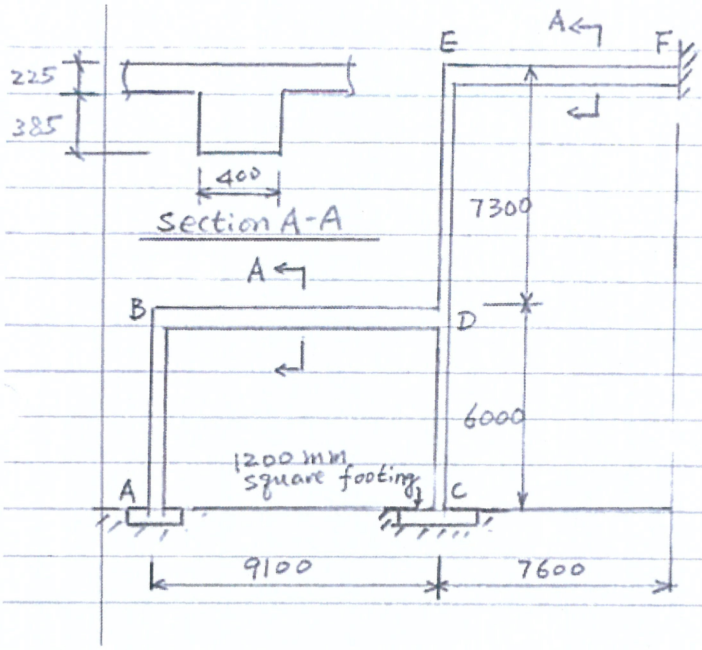
$d'/d = 0.1$

$f_y = 40\ ksi\ (276\ MPa)$

$E_s = 29 \times 10^3\ ksi\ (200000\ MPa)$

Problem 2. (30%)

Design column AB in the nonsway frame shown in the figure. fy = 420MPa, concrete cylinder strength = 20MPa, and the service loads and moments listed below.

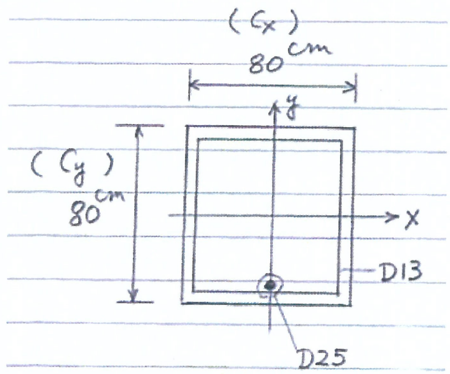


	Column AB
Service Load, P	$P_D = 300\ kN$
	$P_L = 90\ kN$
Service moments at tops of column	$M_D = -70\ kN - m$
	$M_L = -16\ kN - m$
Service moments at bottoms of column	$M_D = -26\ kN - m$
	$M_L = -10\ kN - m$

Problem 3. (40%)

Design the longitudinal reinforcement for a reinforced concrete column subjected to axial compression and biaxial bending. fy = 4200 kgf/cm^2, concrete cylinder strength =280 kgf/cm^2, and the load demands summarized below.

(a) For the design, please use the load contour method and (b) for the review, please use the reciprocal load method to check your design.



(unit: tf and m)	Axial load (tf)	Top of column M_{ux}(tf-m)	Top of column M_{uy}(tf-m)	Bottom of column M_{ux}(tf-m)	Bottom of column M_{uy}(tf-m)	V_u (tf)
Minimum axial load combination	-600.0	-10.0	12.0	108.0	-30.0	30.0

Note: You can make assumptions for each problem that you think are necessary.