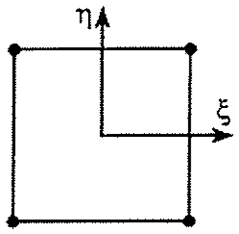
Qualified Examination Spring 2016

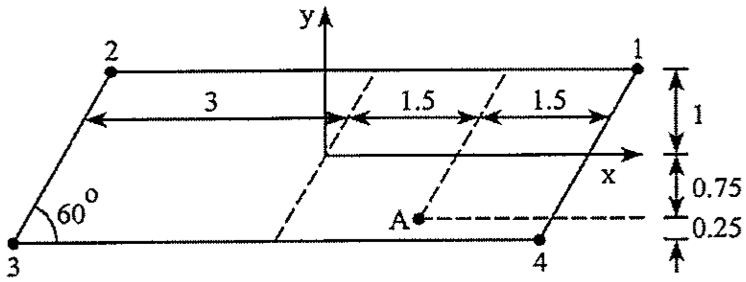
Finite Element Method

(Close book, 100 minutes, 70% to pass)

1. (i) Sketch one of the zero energy mode shape for a 4-node isoparametric plane element with reduced integration rule. (ii) Explain why the strain energy at the reduced integration point is zero? (15%)



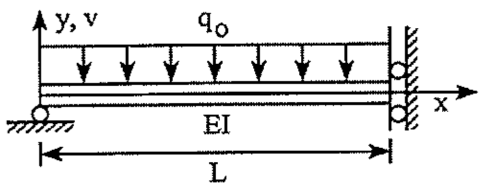
2. A 4-node isoparametric element is shown below. Compute the Jacobian matrix [J] at point A. (15%)



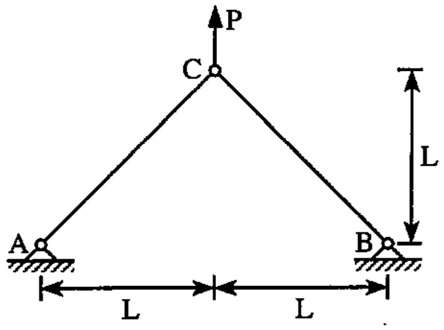
3. Use appropriate Gauss integration rule to obtain the exact solution for the following integral: (20%)

$$I=\int_0^2\int_0^2(3y^2+2x)dxdy$$

4. Consider a beam of constant EI supporting a uniformly distracted load q_o as shown. The differential equation of the beam is $EIv(x)''''+q_o=0$ for $0\le x\le L$, where v(x) is the lateral deflection of the beam. The essential boundary conditions of the beam are $v(0)=0$ and $v'(L)=0$. The nonessential boundary conditions of the beam are $EIv''(0)=0$ and $EIv'''(L)=0$. Assume the approximate deflection of the beam is $\tilde{v}(x)=ax(2L-x)$. Use the Galerkin method to find the generalized d.o.f. a in the approximate deflection. (20%)



5. A 2-bar truss is subjected to a concentrated force P as shown. Assume both bars have the same cross section area A and the same modulus of elasticity E. Use the finite element method to calculate the displacements at node C, the reactions at nodes A and B, and the axial forces in both bars. (15%)



6. The following True-False questions refer to finite elements based on assumed displacements. (15%)

( ) A. The interpolation functions in FEA are almost always trigonometric functions.

( ) B. Within an element, the calculated strains are less accurate than the displacements.

( ) C. The stiffness matrix [K] of a structure is usually unsymmetric.

( ) D. The normal and shear strains in the three-node triangle element are always constants.

( ) E. When a Q4 element is subjected to pure bending, it displays shear strain and bending strain.

Qualifying examination (Elasticity)

(1) Let $\{\boldsymbol{i},\boldsymbol{j},\boldsymbol{k}\}$ be a Cartesian basis, and let

$\boldsymbol{m}_1=(5\boldsymbol{i}+6\boldsymbol{j}-3\boldsymbol{k})/\sqrt{70}$, $\boldsymbol{m}_2=(\boldsymbol{j}+2\boldsymbol{k})/\sqrt{5}$, $\boldsymbol{m}_3=(3\boldsymbol{i}-2\boldsymbol{j}+\boldsymbol{k})/\sqrt{14}$ be three unit vectors. The components of a tensor $\boldsymbol{T}$ in $\{\boldsymbol{i},\boldsymbol{j},\boldsymbol{k}\}$ are

$$\boldsymbol{T}=\begin{bmatrix}1 & 2 & 0\\ 2 & 4 & 0\\ 0 & 0 & 5\end{bmatrix}.$$

Calculate the components of $\boldsymbol{T}$ in $\{\boldsymbol{m}_1,\boldsymbol{m}_2,\boldsymbol{m}_3\}$ (25%)

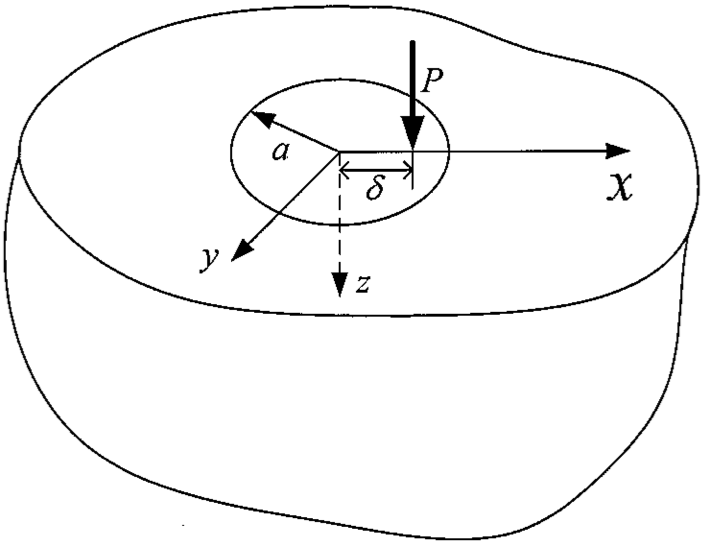
(2) Show that for the case of plane stress, in the absence of body forces, the equations of equilibrium may be expressed in terms of displacements u_1 and u_2 as follows:(15%)(ν : Poisson's ratio)

$$\frac{\partial^2 u_1}{\partial x^2}+\frac{\partial^2 u_1}{\partial y^2}+\frac{1+\nu}{1-\nu}\frac{\partial}{\partial x}\left(\frac{\partial u_1}{\partial x}+\frac{\partial u_2}{\partial y}\right)=0$$

$$\frac{\partial^2 u_2}{\partial x^2}+\frac{\partial^2 u_2}{\partial y^2}+\frac{1+\nu}{1-\nu}\frac{\partial}{\partial y}\left(\frac{\partial u_1}{\partial x}+\frac{\partial u_2}{\partial y}\right)=0$$

Are these equations valid for the case of pane strain? (10%)

(3) A rigid disk is welded to an elastic isotropic half-space. A load, P, is applied to the disk as shown. Considering the most general motion of the disk, state the boundary value problem. (Don't solve the problem. Statement must include equilibrium requirements). (25%)



4. The stress field around a cylindrical hole in an infinite solid, which is subjected to uniaxial tension $\sigma_{11}=\sigma_0$ far from the hole, is given by

$$\sigma_{11}=\sigma_0[1+(\frac{3a^4}{2r^4}-\frac{a^2}{r^2})\cos 4\theta-\frac{3a^2}{2r^2}\cos 2\theta]$$

$$\sigma_{22}=\sigma_0[-(\frac{3a^4}{2r^4}-\frac{a^2}{r^2})\cos 4\theta-\frac{a^2}{2r^2}\cos 2\theta]$$

$$\sigma_{12}=\sigma_0[(\frac{3a^4}{2r^4}-\frac{a^2}{r^2})\sin 4\theta-\frac{a^2}{2r^2}\sin 2\theta]$$

Using the principle of superposition, calculate the stresses near a hole in a solid which is subjected to shear stress $\sigma_{12}=\sigma_0$ at infinity.(25%)