

1. (20%) Let the truss element of Fig. 1 have constant cross-sectional area  $A$  and elastic modulus  $E$ . Generate the element stiffness matrix.

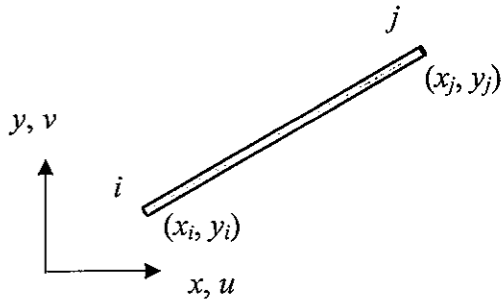


Fig. 1.

2. (30%) The differential equation for the beam problem is

$$EIw_{,xxxx} - q = 0$$

and the nonessential boundary conditions are

$$EIw_{,xx} = \bar{M}$$

$$EIw_{,xxx} = \bar{V}$$

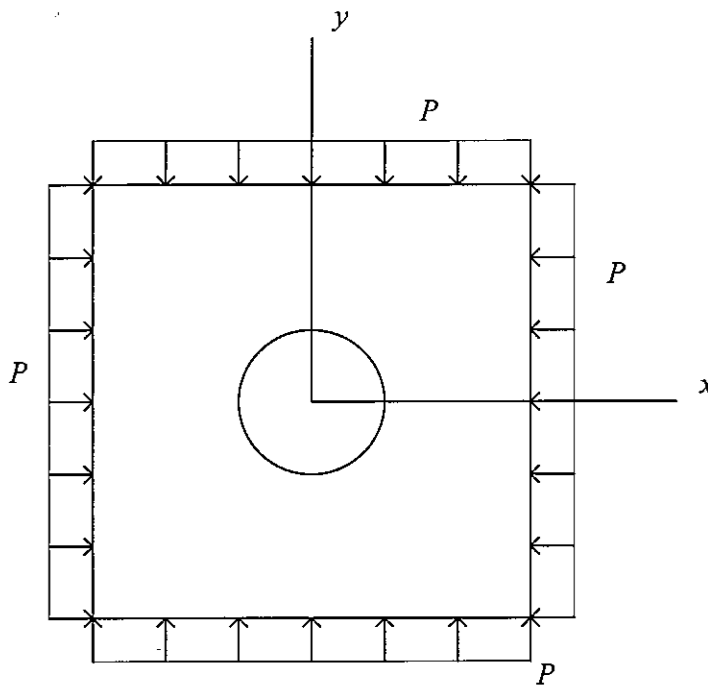
where  $EI$  = bending stiffness,  $w$  = later displacement,  $q$  = distributed loads.

Use Galerkin's method to derive the finite element formulation.

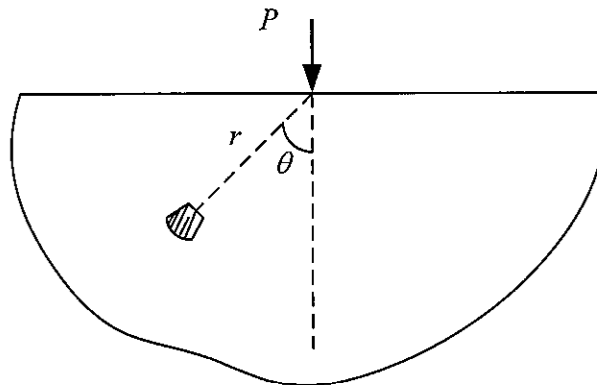
3. (30%) For problem 2, using the nodal displacement  $w$  and rotation  $\theta = w_{,x}$  as the nodal d.o.f. to derive an expression for the element stiffness matrix.
4. (20%) For a simple supported beam of length  $L$ , under uniform load  $q$ , use two element with the stiffness you get in problem 3 to evaluate the displacement of medial point.

Qualifying examination (Elasticity)

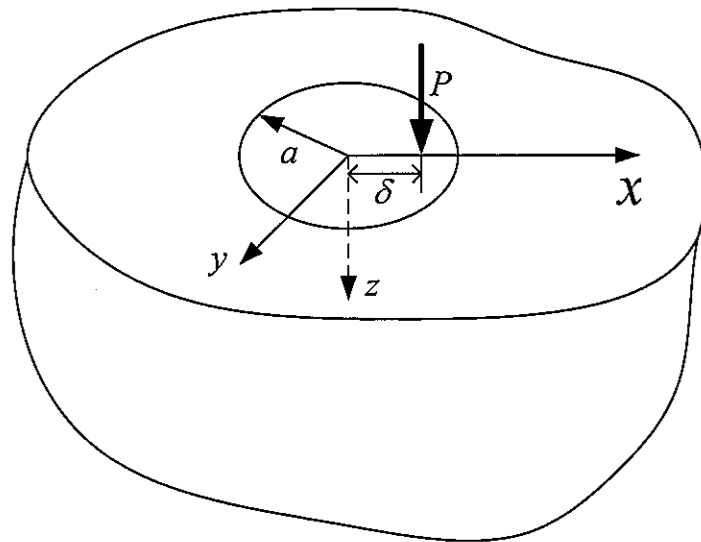
- (1) Determine the stress concentration factor for the 2-D infinite medium with a circular hole in it loaded as shown in Fig. below. The quantity  $P$  is a stress magnitude. (25%)



- (2) Determine the stress field  $\sigma_{rr}, \sigma_{\theta\theta}, \sigma_{r\theta}$  in a semi-infinite linearly elastic isotropic medium due to a concentrated load  $P$  acting on its surface, as shown. (15%)  
Discuss your results. (10%)



- (3) A rigid disk is welded to an elastic isotropic half-space. A load,  $P$ , is applied to the disk as shown. Considering the most general motion of the disk, state the boundary value problem. (Don't solve the problem. Statement must include equilibrium requirements). (25%)



- (4) A solid with volume  $V$  is subjected to a distribution of traction  $t_i$  on its surface. Assume that the solid is in static equilibrium. By considering a virtual velocity of the form  $\delta v_i = A_{ij} y_j$ , where  $A_{ij}$  is a constant symmetric tensor, use the principle of virtual work to show that the average stress in a solid can be computed from the shape of the solid and the tractions acting on its surface using the expression. (25%)

$$\frac{1}{V} \int_V \sigma_{ij} dV = \frac{1}{V} \int_S \frac{1}{2} (t_i y_j + t_j y_i) dA$$

## 103 學年度第 1 學期博士學位候選人資格考試

### 高等瀝青材料學 (20% for each question)

1. Translate the following paragraph.

In the past, the mechanical characterization tests used were generally empirical in nature, but an effort has been made since the middle of 1980s to move towards measuring more fundamental engineering properties. Empirical tests are used less frequently now because they are not able to predict performance for conditions that are different than those under which the tests were developed.

2. Briefly describe the reasons to develop the superpave asphalt binder tests and specifications.
3. Briefly describe the procedures of the Marshall mix design and superpave mix design method.
4. Briefly describe the asphalt refining procedures.
5. Briefly describe the properties of an ideal pavement binder.

# 103學年度第一學期博士學位候選人資格考試

## 工程統計

作答方式：Open Book 考試時間：100分鐘 及格分數：60分

1. A random variable  $Z$  is the sum of two statistically independent random variables  $X$  and  $Y$ , where  $X$  and  $Y$  have the same type of probability distribution but with different parameters.
  - (a) Show three types of probability distribution for  $X$  and  $Y$  so that their sum  $Z$  also has the same type of probability distribution as those for  $X$  and  $Y$ . (15%)
  - (b) For each illustration in (a), state the relationship between the parameters of  $X$ ,  $Y$  and  $Z$ . (15%)
2. Let  $X_1, X_2, \dots, X_n$  be uniformly distributed on the interval 0 to  $a$ .
  - (a) Find the moment estimator of  $a$ . Is this an unbiased estimator? And why? (15%)
  - (b) Find the maximum likelihood estimator of  $a$ . Is this an unbiased estimator? And why? (15%)
3. A multiple regression model is given by  $Z = \alpha + \beta x + \gamma y + \varepsilon$ .
  - (a) Derive the formulae to estimate the parameters  $\alpha$ ,  $\beta$  and  $\gamma$  together with the variance of  $\varepsilon$  if  $n$  observations are obtained. (15%)
  - (b) Determine the estimates for  $\alpha$ ,  $\beta$ ,  $\gamma$  and the variance of  $\varepsilon$  on the basis of the observations as follows. (10%)

$x$	16.7	17.4	18.4	16.8	18.9	17.4
$y$	30	42	47	47	43	41
$z$	210	110	103	103	91	76

4. Assuming that  $X$  is normally distributed, does the following data of  $X$  suggest that the mean of  $X$  exceeds 25? Carry out the test at a significance level of 5%. (15%)

25.8    36.6    26.3    21.8    27.2