Qualified Examination Spring 2014

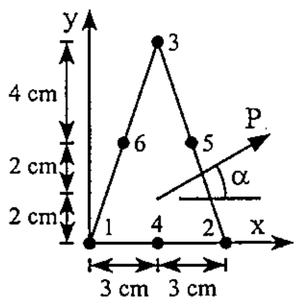
Finite Element Method

(Close book, 100 minutes, 70% to pass)

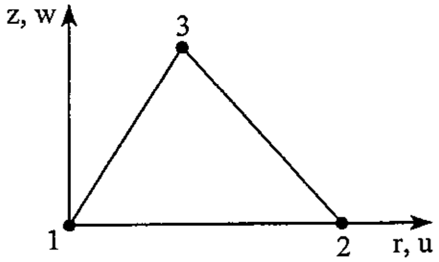
1. The following True-False questions refer to finite elements based on assumed displacements. (15%)
   ( ) A. The interpolation functions in FEA are almost always trigonometric functions.
   ( ) B. Within an element, the calculated strains are less accurate than the displacements.
   ( ) C. The stiffness matrix [K] of a structure is usually unsymmetric.
   ( ) D. The normal and shear strains in the three-node triangle element are always constants.
   ( ) E. When a Q4 element is subjected to pure bending, it displays shear strain and bending strain.

2. (a) What is the parasitic shear? Use the four-node isoparametric plane element with full integration rule to illustrate it. (7%)
   (b) What is the zero-energy mode (hourglass mode)? Use the eight-node isoparametric plane element with reduced integration rule to illustrate it. (8%)

3. The triangular element shown below is subjected to a concentrated force P = 100 N at an angle $\alpha = 30^{\circ}$. Use the area coordinates to calculate the consistent nodal load vector $\{Q\} = \{Q_{1x}, Q_{2x}, Q_{3x}, Q_{4x}, Q_{5x}, Q_{6x}\}^T$. Assume the thickness of the plate is t = 0.5 cm. (20%)



4. A 3-node axisymmetric triangle element is shown below. Let the coordinates of the nodes be (0,0), $(r_2, 0)$, (r_3, z_3), the vector of strains be $\{\varepsilon\} = \{\varepsilon_r, \varepsilon_\theta, \varepsilon_z, \gamma_{zr}\}^T$, and the vector of nodal degrees of freedoms be $\{d\} = \{u_1, w_1, u_2, w_2, u_3, w_3\}^T$. If the relations between the strains and the nodal degrees of freedoms are $\{\varepsilon\}_{4\times1} = [B]_{4\times6}\{d\}_{6\times1}$, find the [B] matrix for the element. (20%)



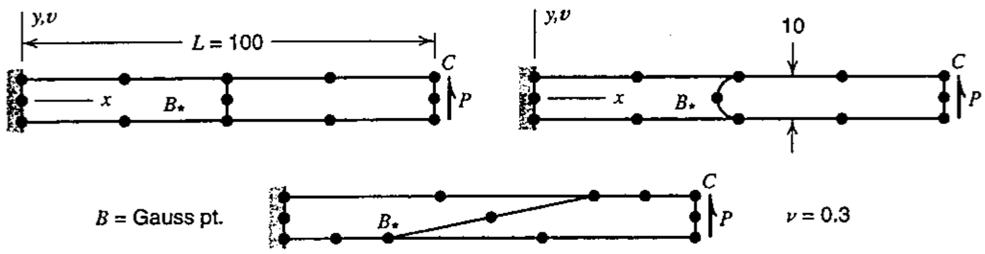
5. (a) Evaluate the following integral by 4-point Gauss quadrature. (10%)

$$I = \int_{-1}^{1}\int_{-1}^{1}\frac{3+\xi^2}{2+\eta^2}d\xi d\eta$$

(b) Is the integral evaluation in part (a) an exact? Why or why not? (5%)

6. A cantilever beam subjected to a concentrated load at the free end is analyzed by 8-node or 9-node isoparametric plane stress elements. The meshes used in the analyses, the normalized stresses σ_{XB}, and the normalized beam deflections at the free end v_C are given below. When the normalized stresses or deflections are equal to 1, it means the numerical result is the same as the exact solution. What kinds of conclusions you can obtain from the results of the FEM analyses? (15%)

STRESSES AND DEFLECTIONS IN TWO-ELEMENT CANTILEVER BEAMS UNDER TRANSVERSE TIP LOAD *P*. EXACT VALUES BY BEAM THEORY ARE UNITY (NEGLECTING TRANSVERSE SHEAR DEFORMATION).



Element type, integration rule	Rectangular elements σ_{xB}	Rectangular elements v_C	Trapezoidal elements σ_{xB}	Trapezoidal elements v_C	Curved sides σ_{xB}	Curved sides v_C
8-node, 2×2	1.000	0.968	0.051	0.362	−0.048	0.430
8-node, 3×3	1.129	0.930	0.048	0.161	0.050	0.221
9-node, 2×2	1.000	1.006	1.125	1.109	0.958	0.955
9-node, 3×3	1.141	0.954	0.687	0.791	0.705	0.737

Elasticity

Department of Civil Engineering, National Cheng Kung University

***Return question-paper after exam**
***Closed-book exam**

1. **(30 points)** Given the relations

$$\sigma_{ij} = s_{ij} + \frac{1}{3}\sigma_{kk}\delta_{ij}, \quad J_2 = \frac{1}{2}s_{ij}s_{ji}, \quad J_3 = \frac{1}{3}s_{ij}s_{jk}s_{ki},$$

where σ and s are symmetric second-order tensors. Show that

(a) $s_{ii} = 0$.

(b) $J_{2,\sigma ij} = s_{ij}$.

(c) $J_{3,\sigma ij} = s_{ik}s_{kj} - 2J_2\delta_{ij}/3$.

2. **(30 points)** For isotropic linear elastic materials, prove the following relations between the elastic moduli E (Young's modulus), G (shear modulus), ν (Poisson's ratio) and k (bulk modulus):

$$\nu = \frac{3k - E}{6k}, \quad k = \frac{GE}{9G - 3E}.$$

Using the fact that for an isotropic body the principal axes of stress and strain coincide and assuming that the stress-strain relation is linear so that superposition holds, derive

$$\varepsilon_{ij} = \frac{1+\nu}{E}\sigma_{ij} - \frac{\nu}{E}\sigma_{mm}\delta_{ij},$$

directly from the definitions of E and ν given by

$$E = \frac{\sigma_{11}}{\varepsilon_{11}}, \quad \nu = -\frac{\varepsilon_{22}}{\varepsilon_{11}} = -\frac{\varepsilon_{33}}{\varepsilon_{11}},$$

in simple tension test.

3. **(30 points)** The elastic complementary energy density U^c is defined viz.

$$U^c \equiv \int_0^{\sigma_{ij}} \varepsilon_{ij} d\sigma_{ij}.$$

Based on this concept, the behavior of an isotropic nonlinear elastic material is described by the following assumed polynomial expression for U^c:

$$U^c\left(I_1, J_2, J_3\right) = aI_1^2 + bJ_2 + cJ_3^2,$$

where a, b and c are constants and I_1, J_2 and J_3 are the stress invariants defined in Problem

1 of this exam.

(a) Derive the stress-strain relations for this material in terms of the constants a, b and c.

(b) Derive the stress-strain relations in simple tension. Find an expression for the tangent Young's modulus E_t defined viz.

$$E_t \equiv \frac{d\sigma}{d\varepsilon},$$

in simple tension, in terms of the stress σ. What is the value of the initial tangent Young's modulus $E_t(0)$ (at $\sigma = 0$)?

(c) Show that the constitutive equations are reduced to those of the isotropic linear elastic material for $c = 0$. Find the relations between the constants a, b, and the elastic moduli E, v for this case.

4. **(10 points)** For isotropic materials, show that the principal axes of strain coincide with the principal axes of stress. Further, show that the principal stresses can be expressed in terms of the principal strains via

$$\sigma_i = 2\mu\varepsilon_i + \lambda\varepsilon_{kk},$$

where σ_i and ε_i are the principal stresses and strains, respectively.

NCKU 1022 Qualify Exam for Ph.D. Candidate
Course: Soil mechanics
Time limitation: 100 min.

Note: There are two pages in the question sheet. Make rational assumptions if necessary

Question 1: briefly explain the following terms: (25 pts)

(a) Flow line
(b) Activity
(c) ϕ=0 concept
(d) Critical void ratio
(e) Proctor test

Question 2: questions related to physical properties of soils; (25 pts)

(a) Plot a three phase diagram and use it to show that $\gamma = \frac{(G_s + eS)\gamma_w}{(1+e)}$ (6 pts)

(b) For a given soil with following properties: $G_s = 2.75$, $\gamma_t = 20.6$ kN/m^3, w=18%, plot the three phase diagram and determine the following properties accordingly: (a) dry unit weight, (b) void ratio, and (c) porosity. (7 pts)

(c) List the physical properties required for USCS soil classification and explain why these properties are involved. (6 pts)

(d) Describe the physical meanings of the three lines on Casagrande plasticity chart. (6 pts)

Question 3: questions related to shear strength of soils: (30 pts)

(a) The state of stress of a soil element is shown in Fig. 1. Plot the Mohr circle, pole, and principle planes. Additionally, calculate the normal and shear stresses on plane AB. (6 pts)

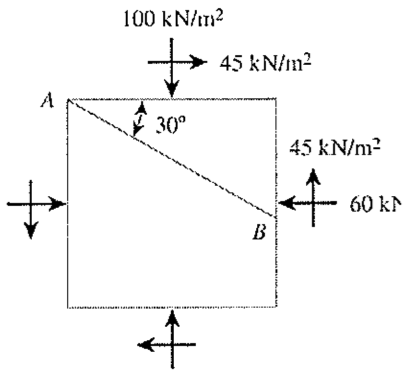


Fig. 1

(b) Define the MIT stress path and express the (m, α) of K_f-line ($q = m + p' \tan\alpha$) in terms of (c', ϕ'). (6 pts)

(c) Plot the stress path of (b) along with the K_f-line. (6 pts)

(d) Describe two laboratory techniques for shear strength of soils and list the advantages and disadvantages between the two techniques. (6 pts)

(e) Plot the typical shearing responses for NC and OC clay in terms of (1) deviator stress vs. axial strain, (2) volumetric strain vs. axial strain, (3) excess pore pressure vs. axial strain, and (4) Mohr circles for total and effective stress. (6 pts)

Question 4: questions related to steady state flow: (20 pts)

(a) A constant head test is conducted on a soil specimen, which is 35 cm long with a cross section area of 125 cm^2 and void ratio of 0.61. The specimen is subjected to 42 cm head difference and 580 cm^3 of water is collected in 3 minutes. Calculate the coefficient of permeability, Darcy's velocity, and seepage velocity. (10 pts)

(b) Calculate the flow rate of unit thickness at downstream for Fig. 2 with $k=2\times10^{-3}$ cm/sec. (5 pts)

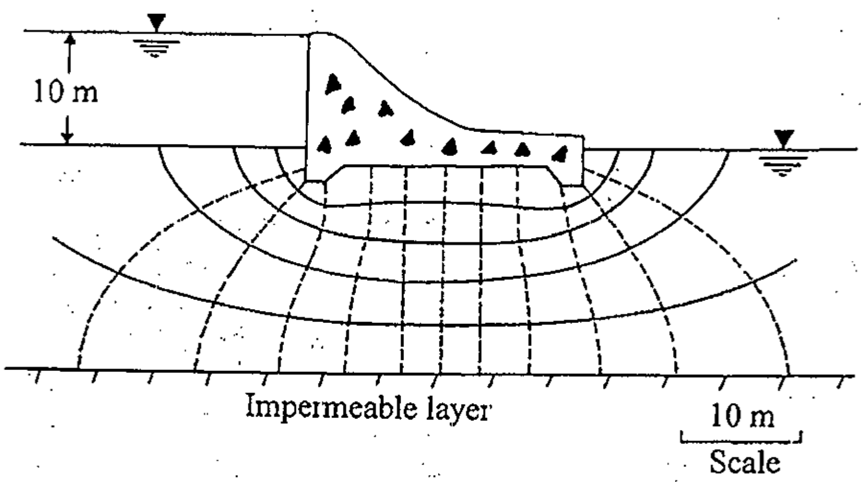


Fig. 2

(c) If the flow net of Fig. 2 is the transformed one for $k_H/k_V = 4$ and $k_V=2\times10^{-3}$ cm/sec, calculate the flow rate of unit thickness at downstream. (5 pts)

102學年度第二學期博士班資格考

考試科目：工程地質

1. 解釋名詞 (20%)

(1) Dip direction? (3%) dip angle? (3%) strike? (3%)

(2) Standard penetration test? (3%) Lugeon test? (3%)

(3) Plot the legend/symbol of normal fault (2%) and syncline (3%) in geological map.

2. Plot the three-dimensional principle stresses of the normal fault, inverse fault and strike-slip fault? (15%)

3. How to design the support system of a tunnel without computer analysis? (15%)

4. What are the main geological and geotechnical factors that must be considered when a dam site is decided? (15%)

5. What do the stereo-net (Wulff net) of rock joints look like for the slope failures of sliding (5%), toppling (5%), wedge failure (5%) and circular failure (5%) ?

6. When we build a very long concrete diaphragm perpendicular to the direction of local groundwater flow to prevent groundwater flowing into a very long underground construction, the concrete diaphragm raises groundwater level at the upstream side and lowers at the downstream side. If the concrete diaphragm is not removed after the construction, what kinds of geotechnical problems that may happen at the upstream and downstream sides, respectively? (15%)

102學年度第二學期博士學位候選人資格考試

工程統計 Engineering Statistics

作答方式：Open Book　考試時間：100分鐘　及格分數：70分

1. The moment-generating function of a random variable X is defined by $G_X(s) = E(e^{sX})$, where E denotes the expectation. The mean and the standard deviation of X are denoted by μ_X and σ_X, respectively.

   (a) Prove $\left.\dfrac{d\ln G_X(s)}{ds}\right|_{s=0} = \mu_X$. (10%)　　(b) Prove $\left.\dfrac{d^2 \ln G_X(s)}{ds^2}\right|_{s=0} = \sigma_X^2$. (10%)

2. Assumed that X is a Poisson variate with the mean occurrence rate of ν, and given the conditional probabilities $P(Y=1|X=1)=p$ and $P(Y=0|X=1)=1-p$. Derive the probability mass function of Y. (20%)

3. The average all-day parking cost in a city may be expressed in terms of the logarithm of the urban population as $E(Y|x) = a + b\ln x$.

   (a) Derive the formulae to estimate coefficients a and b, together with the conditional standard deviation $s_{Y|x}$. (10%)

   (b) Determine the estimates for a, b, and $s_{Y|x}$ on the basis of the observations as follows. (10%)

x	190	310	270	320	460	340	380	520	310	400	470	840	1910	3290	3600
y	0.50	0.48	0.53	0.58	0.60	0.67	0.69	0.75	0.80	0.80	0.81	0.92	0.92	1.40	1.12

   (c) The all-day parking cost depending on the population is hypothesized to follow a normal distribution. Perform the χ^2-test for goodness of fit at the significance level of 5%. (10%)

4. The following observations are the axial stiffness indices (kips/in) for plate lengths 4, 6, 8, 10, and 12 in. Does variation in plate length have any effect on true average axial stiffness at the significance level of 1%? Display your results in an ANOVA table. (30%)

4	309.2	409.5	311.0	326.5	316.8	349.8	309.7
6	402.1	347.2	361.0	404.5	331.0	348.9	381.7
8	392.4	366.2	351.0	357.1	409.9	367.3	382.0
10	346.7	452.9	461.4	433.1	410.6	384.2	362.6
12	407.4	441.8	419.9	410.7	473.4	441.2	465.8

102學年度第二學期博士學位候選人資格考試 Advanced Bituminous Materials

高等瀝青材料學 (20% for each question)

1. Translate the following paragraph.
   For the past fifty years or more, pavement design agencies have always pressed for an immediate answer to their needs and problems. Long-term, well considered sequential efforts usually have been rejected because of the time and expense involved. The effect has been that structural and mixture design procedures have stagnated, and an integrated design and analysis system have not evolved.

2. Briefly describe the asphalt cement grading system. (including PEN, AC, AR, and SHRP)

3. Briefly describe the aggregate properties tests for the Superpave mix design method.

4. Briefly describe the hot mix asphalt facilities.(including the drum mix facility and batch facility)

5. Briefly describe the properties of an ideal pavement binder.