# Finite Element Method

(Close book, 100 minutes, 60% to pass)

1. For the plane truss shown below, let the structural displacement vector be $\{D\}=\{u_a,v_a,u_b,v_b,u_c,v_c,u_d,v_d\}^T$. (i) Form the structural stiffness matrix $[K]_{8\times8}$. (ii) Determine the displacements at nodes a and b. (iii) Calculate the reactions at nodes b, c and d. (iii) Find the axial force in member ac. Let E = 200 GPa, $\alpha=30^O$, $\beta=45^O$ and $\gamma=40^O$. Assume A_{ab}=200 cm^2, A_{bc}=180 cm^2, A_{ac}=150 cm^2, and A_{ad}=200 cm^2. (25%)
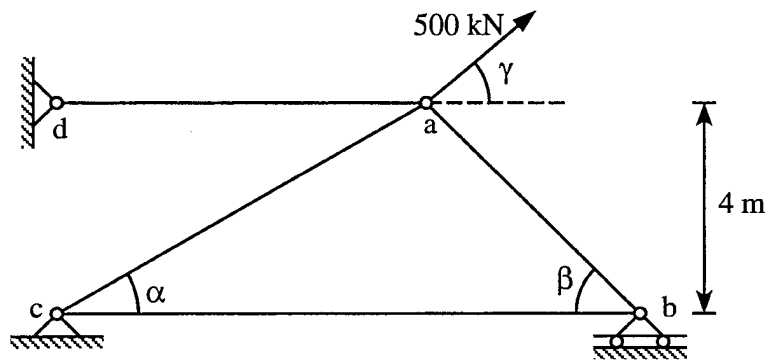
2. The potential energy of a beam-column subjected to axial compressive force P and restrained by translational spring with spring constant k at x = L is

$$\Pi_p = \frac{1}{2}\int_0^L EI\left(\frac{d^2v}{dx^2}\right)^2 dx - \frac{P}{2}\int_0^L\left(\frac{dv}{dx}\right)^2 dx + \frac{1}{2}kv_L^2$$

where $v_L = v(L)$. Using the calculus of variation to find the governing differential equation and nonessential boundary conditions of the beam. [Hint: The essential boundary conditions of the beam is $v(0)=0$ and $v'(0)=0$] (20%)
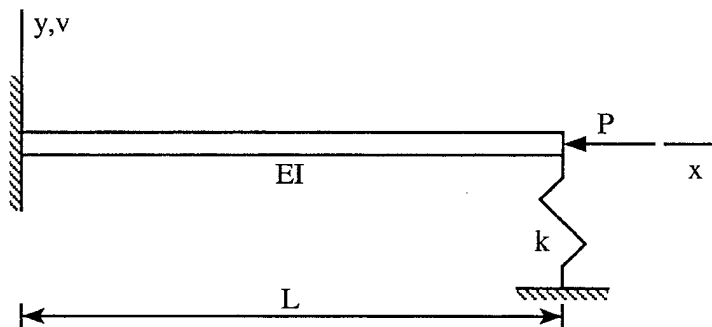
3. A 4-node isoparametric element with thickness $t = 0.1$ is shown below.

(i) Compute the Jacobian matrix [J] and its determinant $|J|$ at the centroid ($\xi = 0, \eta = 0$) of the element. (10%)

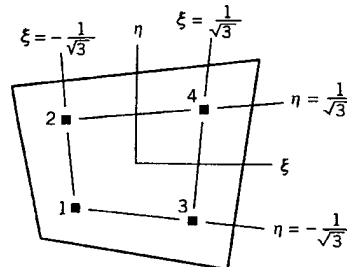(ii) Compute the strains ε_x and γ_{xy} at the centroid of the element if the nodal displacements are $\{d\} = \{u_1, v_1, u_2, v_2, u_3, v_3, u_4, v_4\}^T = \{3, 1, 3, -1, 3, 2, 3, 1\}^T$. (10%)
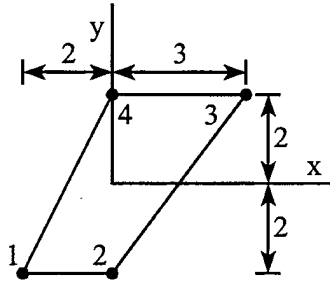

4. Use a 2×2 Gauss integration rule to evaluate the following integral: (15%)

$$I = \int_3^6 \int_{-1}^2 (2 + \frac{y}{1 + 3x} + 4x^2) dy\, dx, \quad (3 \le x \le 6, -1 \le y \le 2)$$

Is this integral evaluation an exact solution? Why or why not? (5%)

5. A cantilever beam subjected to a concentrated force at the free end is modeled by 20-node solid elements. From the result of the analysis, the deformed shapes of the cross sections near A and B are shown below. (i) What phenomenon is exhibited in the deformation shape of the beam? (ii) In your judgment, is the result of this finite element analysis accurate? Why or why not? (15%)

Cross section near A

Cross section near B
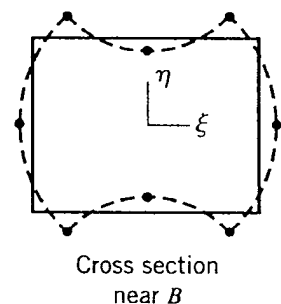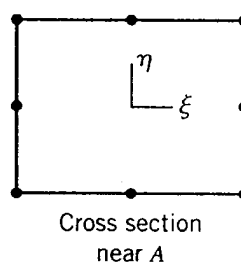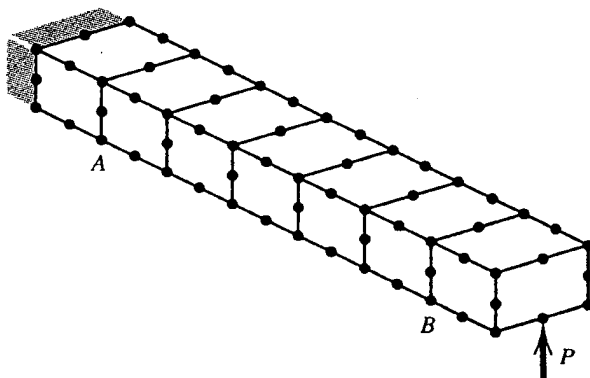
九十八學年度第二學期博士學位候選人資格考試
結構動力學試題

1. (30%) Consider the two-degree system and the modal parameters shown in fig. 1. Determine (a) the equations of motion for the system, (b) the natural frequencies and characteristic shapes, and (c) write the modal equations of motion.

2. (40%) For an undamped one-degree system with sinusoidal support motion shown in Fig. 2. Determine the spectrums of the maximum relative motion of the mass with respect to the support and the maximum absolute acceleration of the mass. Note that the support acceleration continues for only one cycles.

3. (30%) Consider an idealized vehicle traveling over a simply supported bridge shown in Fig. 3 at constant speed v_c. (a)Derive the equation of motion for the system. (b) Assume that the deflection shape of the bridge is $\sin(\pi x/L)$, derive the equation of motion for the vertical motion of the vehicle. (c) Find the maximum vertical motion of the vehicle.

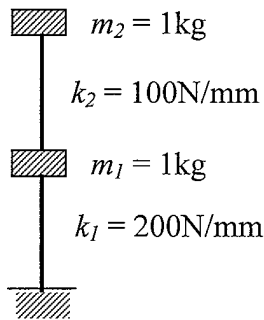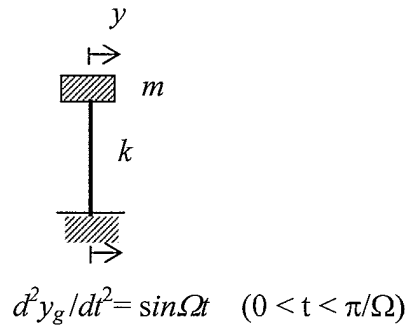$m_2 = 1\text{kg}$
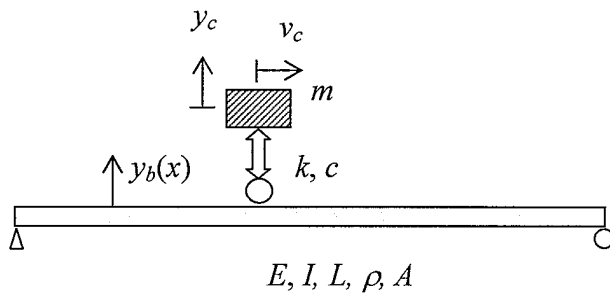
$k_2 = 100\text{N/mm}$

$m_1 = 1\text{kg}$

$k_1 = 200\text{N/mm}$

Fig. 1.

y

m

k

$d^2 y_g/dt^2 = \sin\Omega t \quad (0 < t < \pi/\Omega)$

Fig. 2.

y_c $\quad v_c$

m

$y_b(x)$ $\quad k, c$

E, I, L, ρ, A

Fig. 3.

(1) Show that the equilibrium equations for an isotropic linearly elastic medium with Lame constants λ, μ are expressible in the form

$$(\lambda + \mu)u_{j,ji} + \mu u_{i,jj} = 0$$

in the absence of body force(10%). Show that these equations are satisfied identically when $\mathbf{u}$ has the Papkovich-Neuber representation

$$u_i = \psi_i + \alpha(\phi + x_j\psi_j)_{,i}$$

where the functions ϕ, ψ_i are harmonic, so long as α takes an appropriate value. What is the value of α ? (15%)

(2) Torsion of a uniform, isotropic, linearly elastic prismatic cylinder, whose generators are parallel to the x_3 axis, generates a displacement field of the form

$$u_1 = -\beta x_2 x_3 , \; u_2 = \beta x_1 x_3, \; u_3 = \beta\, w(x_1, x_2)$$

Show that the only independent non-zero stress components are σ_{13}, σ_{23} (10%).

Show that equilibrium is satisfied if

$$\sigma_{13} = \mu\beta\psi_{,2}, \quad \sigma_{23} = -\mu\beta\psi_{,1}$$

for some stress function $\psi(x_1, x_2)$. Here μ is the shear modulus. (10%) By

relating σ_{13}, σ_{23} to the strain field, deduce that

$$\psi_{,11} + \psi_{,22} = -2 \; (5\%)$$

(3). At a point, the traction vector on three planes are obtained as follows

$$\mathbf{t}_n = \mathbf{e}_1 + 2\mathbf{e}_2 + 3\mathbf{e}_3 \qquad \text{for} \qquad \mathbf{n} = -\mathbf{e}_1$$

$$\mathbf{t}_n = -2\sqrt{3}\mathbf{e}_1 + \frac{2}{3}\sqrt{3}\mathbf{e}_2 \qquad \text{for} \qquad \mathbf{n} = \frac{1}{\sqrt{3}}(\mathbf{e}_1 + \mathbf{e}_2 + \mathbf{e}_3)$$

$$\mathbf{t}_n = 2(-\mathbf{e}_1 + \mathbf{e}_2 + \mathbf{e}_3) \qquad \text{for} \qquad \mathbf{n} = \mathbf{e}_2$$

Find the stress tensor $\boldsymbol{\sigma}$ at that point.   (25%)

(4) The constitutive law for isotropic linearly elastic medium is

$$\sigma_{ij} = \lambda \varepsilon_{kk} \delta_{ij} + 2\mu \varepsilon_{ij}$$

where λ, μ are Lame constants. Use this form to derive the representations for E (Young's modulus) and ν (Poisson's ratio) in terms of λ and μ.(25%)

98 學年度第二學期博士班資格考

科目:土壤力學

考試時間: 100 min　　Soil Mechanics

(附註: 若條件不足請自行合理假設並詳列解題過程)

一、名詞解釋 (20 %):

1. Compaction curve
2. Quick clay
3. Mohr-Coulomb failure criterion
4. Critical hydraulic gradient

二、回答下列有關土壤基本物理性質之問題 (20 %):

1. 以三相圖推導 $\gamma_d = \dfrac{G_s \gamma_w}{(1 + \dfrac{G_s w}{S})}$。

2. 說明 Casagrande 之 plasticity chart 其 A-line 與 U-line 之意義。
3. 列出 USCS 土壤分類所需進行之試驗項目及各試驗之目的。
4. 說明 Atterberg limits 及其意義。

三、回答下列有關滲流之問題 (20 %):

1. 說明圖一之流線網其對應之控制方程式、邊界條件及重要假設。
2. 若 $k_x = k_z = k$，計算此一水工結構物之滲流量(quantity of seepage loss)。
3. 若 $k_x = 2k, k_z = k$，計算此一水工結構物之滲流量。
4. 計算 a~e 點之上浮壓力(uplift pressure)並估算 g 點之水力梯度。
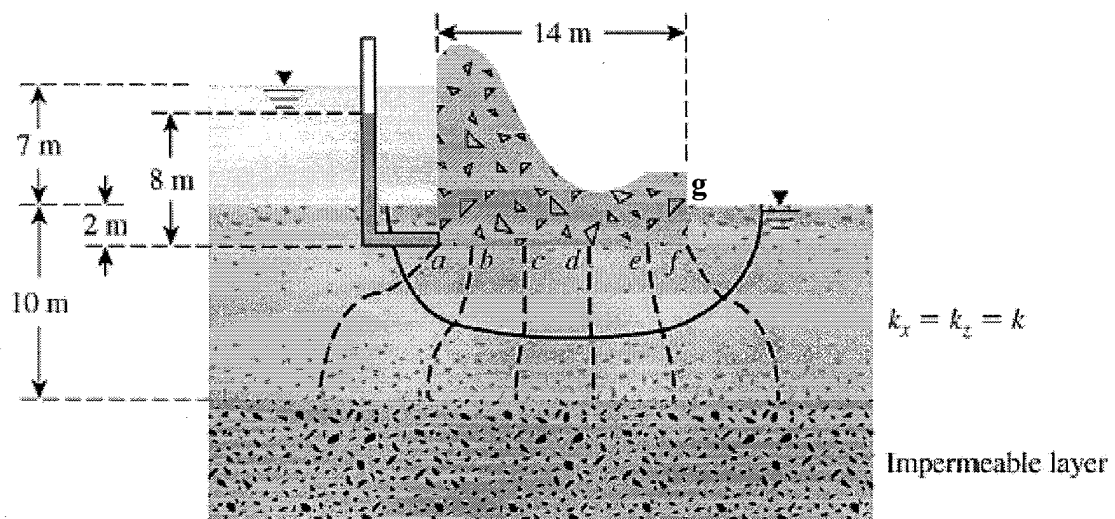
圖一

四、回答下列有關壓密之問題 (20 %):
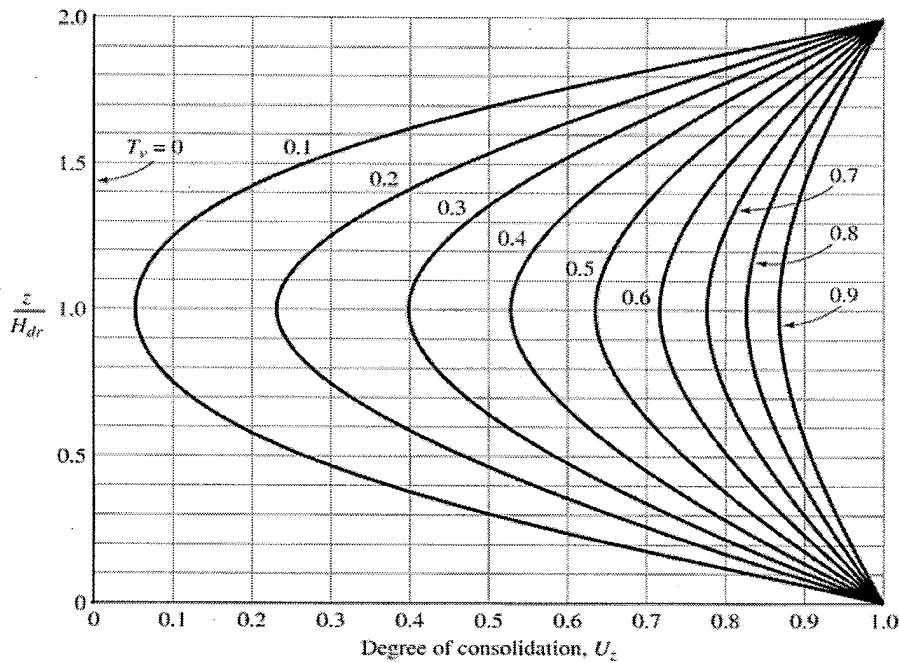
1. 圖二為某黏土層之 progress of consolidation，請寫出適用於此圖之邊界與初始條件。

2. 若該黏土層壓密係數(coefficient of consolidation) $c_v = 8 \times 10^{-8} \, m^2/s$，土層厚度為 10 m，計算 5 年後在深度 0、2.5、5、7.5 及 10 m 之壓密度。

3. 若該黏土層所受之平均垂直有效應力增量為 100 kPa，計算 5 年後在上題 5 個深度之殘餘超額孔隙水壓力。

4. 若該土層受 100 kPa 平均垂直有效應力增量之總沉陷量為 1.0 m，由上二題結果估算在 5 年時之地表沉陷量。

$$T_v = \frac{c_v t}{(H_{dr})^2}$$

圖二

五、回答下列有關土壤剪力強度之問題 (20 %):

1. 某一土壤其 Mohr-Coulomb 剪力強度參數為(c', φ')，請推導於破壞時其主應力具有下列關係: $(\sigma_1 - \sigma_3)_f = \dfrac{2c \cos\phi + 2\sigma_3 \sin\phi}{1 - \sin\phi}$。

2. 一砂土進行 SCD 三軸試驗，其反水壓為 100 kPa，等向壓密應力為 100 kPa，破壞時最大主應力為 400 kPa，請繪出其飽和、壓密及破壞時之有效應力莫爾圓圖(Mohr diagram)，並計算此砂土之摩擦角及破壞時之破壞面。

3. 繪出上題土壤之有效應力路徑並標示其 K_f-line。

2

九十八學年度第二學期博士學位候選人資格考試

考試科目: 工程地質

一. 解釋名詞 (39%)

　　(1) 何謂震央?震源?震度?地震規模?(8%)

　　(2) 何謂節理?何謂斷層?(4%)

　　(3) Strike? Dip? Dip direction? (12%)

　　(4) 標準貫入試驗? Lugeon test? (6%)

　　(5) 褶皺軸脊?請畫出背斜與向斜之地質符號?(9%)

二. 請繪出包溫式反應系列,並標出相對應結晶溫度與抗風化性變化(12%)

三. 試述岩石地層單位分類依據,及其分類所使用的單位並各舉各單位一例(12%)。

四. 請定義活動斷層(15%)

五. 請分別敘述三種變質岩及其變質前的母岩(12%)

六. 試述法國 Malpasset 水庫案例與義大利 Vaiont 水壩分別對水庫工程地質所造成的影響?(10%)

鋼鐵材料與結構   2010.3.26

考試方式: **Closed Book**

考試時間: 分鐘

1. (40%) Please give detailed explanations for the following questions:

    (a) What are the full English and Chinese names of the four types of welding, SMAW, FCAW, GMAW and ESW?

    (b) Why is it important to have the limitations for minimum size of fillet weld in fillet weld design?

    (c) What are the differences between LRFD & ASD in steel design?

    (d) Please describe how the four alloy elements, nickel (Ni), chromium (Cr), molybdenum (Mo) and vanadium (V), affect the properties of steel.

   (e) Please explain the effect of residual stress on steel column strength.

2. (15%) Please give a detailed explanation for the approximate second-order analysis (P-delta analysis) procedure used for the design of frames in LRFD steel design. (Note: except for showing the formulas, you need to give a simple example to demonstrate how to use this second-order analysis procedure)

3. (25%) Consider the following simply-supported beam-column with a triangular loading shown in the following figure.

(1) Please derive the y-direction deflection formula $v(x)$ for this beam-column.

(2) Please derive the moment formula $M(x)$ for this beam-column

(3) Please obtain the theoretical moment magnification factor MAF (or B_1) for this beam-column.
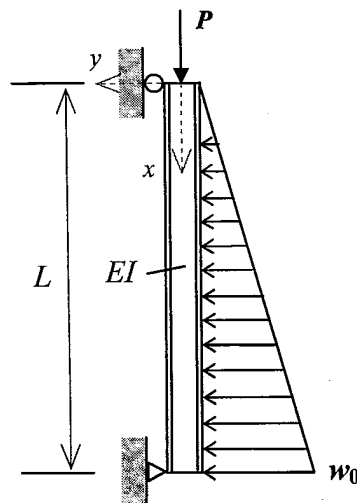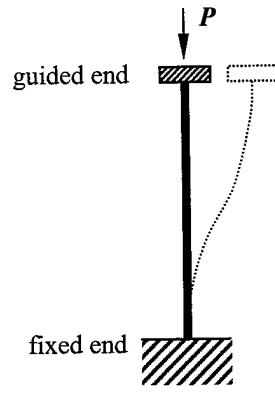

Note: Assume linearly-elastic material

4. (20%) Please derive the theoretical effective length factors (K) for the following two columns.

free end

guided end

fixed end

fixed end
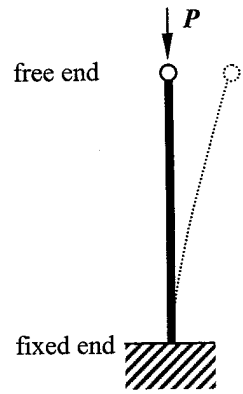
Column (I)

Column (II)

材料機械性質

1. If the compliance matrix of elasticity is given by

$$\begin{bmatrix} S_{11} & S_{12} & S_{13} & 0 & 0 & 0 \\ & S_{11} & S_{13} & 0 & 0 & 0 \\ & & S_{33} & 0 & 0 & 0 \\ & & & S_{44} & 0 & 0 \\ & symm & & & S_{44} & 0 \\ & & & & & 2(S_{11}-S_{12}) \end{bmatrix},$$

what are the planes of material symmetry? What kind of crystal structures would it correspond to? Also, derive the elastic modulus in $<110>$ direction for such a material. (15%)

2. Explain two yield criteria, Tresca and Von Mises, and show their graphical representation for plane stress respectively. Will hydrostatic stresses affect the prediction by Tresca or Von Mises criterion? Give the reason for each criterion. (15%)

3. Write three fracture criteria of single crack and explain each criterion. If material is ductile, what will happen near crack tip? How does the ductility affect fracture criteria in general? (15%)

4. "The elastic deformation on loading and the recovery of stain on unloading involves the stretching of atomic bonds." Would this statement be true of the large elastic deformation observed in rubbers? Why or why not? Also, how does the molecular weight influence the elasticity of rubbers? (15%)

5. Explain "Time-temperature" equivalence in polymeric materials. (10%)

6. In metals, the yield strength is observed to be

$$\sigma_s = A + Bd^{-1/2}$$

Where A, B are constants. Explain the parameter d based on the mechanism of plasticity of metal at low temperature. What is the physical interpretation for constant A? (15%)

7. Briefly explain in terms of the physical mechanisms of creep for metal and polymers, respectively. Also explain the effects of grain size, stress and temperature on the trends in creep behavior of metal. (15%)

1. What are update anomalies, addition anomalies, and deletion anomalies within the data redundancy? (20%)

2. What is VPN(Virtual Private Network)? How can we apply VPN to the construction industry? (10%)

3. What is OLAP (Online Analytical Process)? If you were a material supplier how can the OLAP system help you develop marketing strategies? (10%)

4. Use the following dependencies to normalize Table 1 to 3rd normal tables. (20%)
    $(A, B) \rightarrow C, D, E, F, G$
    $B \rightarrow D, E, F$
    $C \rightarrow B$
    $E \rightarrow F$

    Table 1

A	B	C	D	E	F	G

5. Use the following business rules to answer the questions:
    a. A department employs many employees, but each employee is employed by one department.
    b. Some employees, know as "Rover," are not assigned to any department.
    c. A division operates many departments, but each department is operated by one division.
    d. An employee may be assigned to many projects and a project may have many employees assigned to it.
    e. A project must have at least one employee assigned to it.
    f. One of the employees manages each department.
    g. One of the employees runs each division.

    5.1. Develop an appreciate E-R diagram (20%)
    5.2. Write all the cardinalities into the model.(10%)
    5.3. Modify the E-R model by splitting the M:N relationship into two 1:M relationships that are connected through a composite entity. Then rewrite the connectivities and cardinalities to match the changes you have made.(10%)

工程時程控制　博士資格考　99 年 3 月

一、 請說明浮時擁有權(ownership of float)的觀念及其演進 (20 分)

二、 請說明進度規劃與控制的方法與過程，從拿到一個專案開始，分工結構、估價、作業、關係、排程、調整、進度更新、完成百分比、超前或落後、預測、修正行動等。(20 分)

三、 大工程常延誤(如高鐵通車)，你覺得可能的原因出在哪裡? 爲甚麼? (20 分)

四、 何謂 CPI (cost performance index), SPI (schedule performance index), EAC (estimate at completion)？如何計算？請說明產生過程。(20 分)

五、 某工作有下列作業與關係，請針對作業不分裂(split)，計算各作業最早、最晚時間，劃出網圖，指出要徑。(20 分)

作業	工期	後續作業	關係延時
A	12	B	FF5
B	10	D	SF2,10
D	12	E	FS0
E	9	F	FF7
F	11	G	SS8
G	10		

Please answer in English or Chinese. Answer all the questions concisely.

The traveling salesman problem (TSP, 旅行推銷員問題) is deeply related with the minimum spanning tree problem (MSTP, 最小覆蓋樹問題). This set of questions will explore part of the common roots shared by these two important problems. Please refer to the following definition.

Definition (Held and Karp, 1970): A 1-tree is a network with nodes 1, 2, ..., n, where the nodes 2, 3, ..., n form a tree, and node 1 is attached to the tree by two arcs.

1. (10%) Please define the TSP, and describe a method to solve for its true optimal solution. Comment (do not prove) on its complexity.

2. (10%) Please define the MSTP, and describe a method to solve for its true optimal solution. Comment (do not prove) on its complexity.

3. (10%) Prove that a feasible solution to a TSP is a 1-tree.

4. (20%) Please suggest a method to solve for a minimum-cost 1-tree for a given network. Comment (do not prove) on the complexity of your method.

5. (20%) Prove the following statement: if a minimum-cost 1-tree is a TSP tour, then it is also an optimal solution to the TSP.

6. (20%) Consider a network G of n nodes, and let π_i be a real value associated with node i. Let c_{ij} be the cost of arc (i,j), and let $\bar{c}_{ij} = c_{ij} + \pi_i + \pi_j$. Prove that the optimal TSP solution with respect to c_{ij} is also optimal with respect to $\bar{c}_{ij}$.

7. (10%) Does the property stated in the previous question apply to the MSTP? Why?