## 105學年度第2學期土研所博士班資格考試(106.03)

## 有限元素法

(1) A second-order ordinary differential equation with variable coefficients and its corresponding boundary conditions are given as follows.

GE: $$\frac{d}{dx}\left(a(x)\frac{du(x)}{dx}\right)+b(x)=0 \qquad \text{in } 0<x<1 \qquad (1a)$$

BCs: $$u(x=0)=0, \text{ and } \left. a\frac{du}{dx}\right|_{x=1}=0; \qquad (1b)$$

where $u(x)$ is the unknown function; $a(x)$ and $b(x)$ are given functions.

(a) State a physical problem of which governing equation and possible boundary conditions refer to Eqs. (1a) and (1b). (10%)

(b) Construct the weak form with respect to the strong form (i.e. Eqs. (1a) and (1b)). (10%)

(c) Let $a(x)=1$ and $b(x)=\sin(\pi x)$, and determine the exact solution of $u$(x). (10%)

(d) Let $a(x)=1$ and $b(x)=\sin(\pi x)$. Compute the finite element solution using two linear elements with a uniform mesh. (10%)

(e) Compare the solutions of $u(x)$ and d$u(x)$/dx obtained by the FEM and the analytical method. (10%)

(2) If the nodal values of the element shown in Fig. 2 are $u_i=\hat{u}_i\ (i=1,2,3)$, compute $u$, $\partial u/\partial x$ and $\partial u/\partial y$ at point $(x,y)=(0.325, 0.325)$. (25%)

Hint: The linear interpolation functions $\phi_i^{(e)}(x,y)=\frac{1}{2A_e}\left(\alpha_i^{(e)}+\beta_i^{(e)}x+\gamma_i^{(e)}y\right)\ (i=1,2,3)$ and $\alpha_i^{(e)}=x_j y_k - x_k y_j$, $\beta_i^{(e)}=y_j-y_k$, $\gamma_i=-(x_j-x_k)$, ($i\neq j\neq k$; $i,j$ and $k$ permute in a natural order).

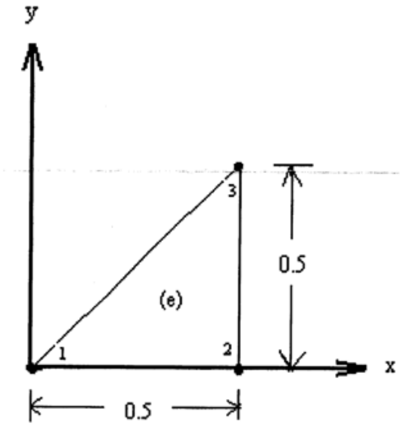



Fig. 2

(3) Show that the bilinear interpolation functions for the four-node triangular element in Fig. 3 are of the form (25%)

$$\phi_i^{(e)} = A_i + B_i \xi + C_i \eta + D_i \xi \eta \quad (i=1, 2, 3, 4)$$

where $A_1 = 1$, $A_2 = A_3 = A_4 = 0$, $-B_1 = B_2 = 1/a$, $B_3 = B_4 = 0$,

$$C_1 = \frac{6ab - a^2 - 2b^2}{ac(a-2b)}, \quad C_2 = \frac{2b(a+b)}{ac(a-2b)}, \quad C_3 = \frac{a+b}{c(a-2b)}, \quad C_4 = \frac{-9b}{c(a-2b)}$$

$$D_1 = D_2 = D_3 = -\frac{1}{3}D_4 = -\frac{3}{c(a-2b)}.$$

Hint: The coordinate of point 4 is $\left(\frac{a+b}{3}, \frac{c}{3}\right)$.

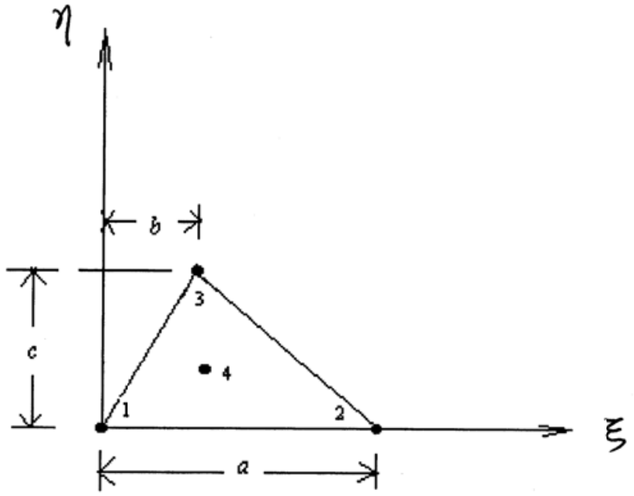



Fig. 3

# 105學年度第二學期博士學位候選人資格考試

高等瀝青材料學 (20% for each question)

1. Explain the following paragraph.
   Empirical tests are used less frequently now because they are not able to predict performance for conditions that are different than those under which the tests were developed. These tests were never good at predicting performance and have become less reliable in prediction performance as axle loads and tire pressures have continued to increase.

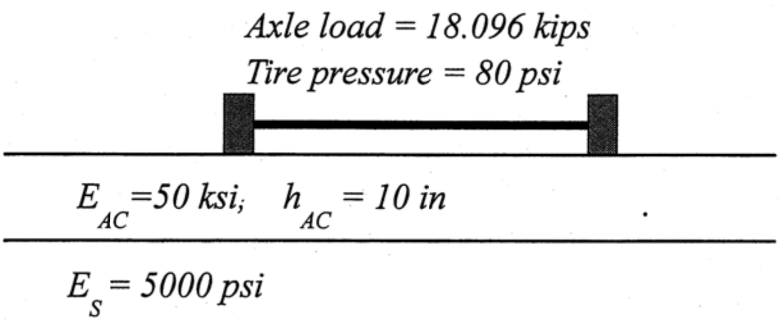
2. Briefly describe the asphalt refining procedures.

3. Briefly describe the major superpave asphalt binder testing equipment and purpose.

4. Briefly describe the procedures of the Marshall mix design and superpave mix design method.

5. Briefly describe the properties of an ideal pavement binder.

1. Please determine the maximum deflection and maximum vertical stress in subgrade. **(2x10%)**




2. Use Westergaard solution to analyze the maximum stresses of a concrete slab (150in×210in×14in) resting on Winkler foundation ($k$=200 psi/in) and loaded with a single wheel, the results are different from the results of finite element analysis by 15%.
   (a) calculate the maximum tensile stress in the concrete slab. **(5%)**
   (b) calculate the maximum deflection in the concrete slab **(5%)**
   (c) calculate the maximum compressive stress in the subgrade. **(5%)**
   (d) list the possible causes of the differences from FEM results. **(5%)**

3. According to AASHO Road Test, a flexible pavement with SN=4 subjected a single-axle load of 20 kip and a tandem-axle load of 40 kip.
   (a) Based on a $p_t$=2.5, determine which axle is more destructive to the pavement. **(10%)**
   (b) Try to explain the mechanism causes result in (a) **(10%)**

4. Please show the computation of stress invariant in the table below. (10%), and determine the type of the material (granular or fine-grained) **(10%)**

| Confining pressure $\sigma_3$ (psi) | Deviator stress $\sigma_d$ (psi) | Recoverable deformation (0.001 in.) | Recoverable strain $\epsilon_r$ ($\times 10^{-3}$) | Resilient modulus $M_R$ ($\times 10^3$ psi) | Stress invariant $\theta$ (psi) |
|---|---|---|---|---|---|
| 20 | 1 | 0.264 | 0.066 | 15.2 | 61 |
| | 2 | 0.496 | 0.124 | 16.1 | 62 |
| | 5 | 1.184 | 0.296 | 16.9 | 65 |
| | 10 | 2.284 | 0.571 | 17.5 | 70 |
| | 15 | 3.428 | 0.857 | 17.5 | 75 |
| | 20 | 4.420 | 1.105 | 18.1 | 80 |

5. Please explain the theoretical and algorithm differences of backcalculation between flexible pavement and rigid pavement. **(20%)**

# 105學年度第二學期博士學位候選人資格考試

# 工程統計

作答方式：Open Book　考試時間：100分鐘　及格分數：60分

1. There are three transportation ways from city A to city B, namely, by highway, rail, and air. Assume that transportation times follow normal distributions, and their means (in hour) and coefficients of variation, $(\mu, \delta)$, by highway, rail, and air are $(2.5, 20\%)$, $(2.2, 10\%)$, and $(2, 15\%)$, respectively. For each travel from city A to city B, a passenger has the chance of 50% by highway, 40% by rail, and 10% by air due to his budget.
   (a) Obtain the probability of transportation time from city A to city B more than 2.5 hours by each transportation way. (10%)
   (b) For one travel from city A to city B, obtain the probability of the passenger's transportation time more than 2.5 hours. (10%)
   (c) If the passenger takes an average of 15 travels from city A to city B in a year, what is the probability of his travel with transportation time more than 2.5 hours less than and equal to one time in a month? Give the assumption on your answer. (5%)

2. Let $X_1, X_2, \ldots, X_n$ be a random sample from a Rayleigh distribution with parameter $\lambda$ in the following:

$$f_X(x) = \frac{x}{\lambda^2}\exp\left[-\frac{1}{2}\left(\frac{x}{\lambda}\right)^2\right], \quad x \geq 0$$

   (a) Find the maximum likelihood estimator of $\lambda$. (10%)
   (b) Is the estimator obtained in (a) unbiased? And why? (5%)

3. Given a sample of $n = 9$ as follows:
   0.814, 0.838, 0.852, 0.809, 0.875, 0.789, 0.864, 0.829, 0.844
   (a) Obtain a 95% confidence interval for the mean value. (15%)
   (b) The null and alternative hypotheses are

$$\begin{cases} H_0 : \mu = 0.85 \\ H_1 : \mu \neq 0.85 \end{cases}$$

   Test the hypothesis at the 5% significance level. (15%)
   (c) Give your assumptions in answering (a) and (b). (5%)

4. An empirical formula for a variable $Z$ depending on variables $X$ and $Y$ is proposed as follows:

$$Z = \alpha X^{\beta} Z^{\gamma}$$

   The observed data is given by $(x_i, y_i, z_i), i = 1, 2, \ldots, n$.
   (a) Find the formula for the **sum of squared errors** in order to estimate coefficients $\alpha$, $\beta$, and $\gamma$ on performing the **linear** regression of $\ln Z$ on $\ln X$ and $\ln Y$. (15%)
   (b) Use the results of (a) to obtain the formula in estimating the conditional variance $Var(\ln Z | x, y)$. (10%)